Time Series Analysis

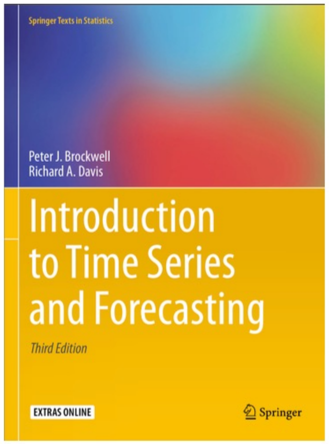
INFO 523 - Lecture 11

Dr. Greg Chism

Lesson 1:

Understanding Timeseries

References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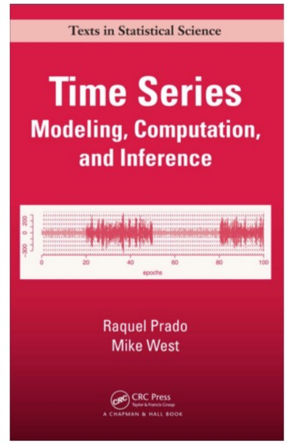
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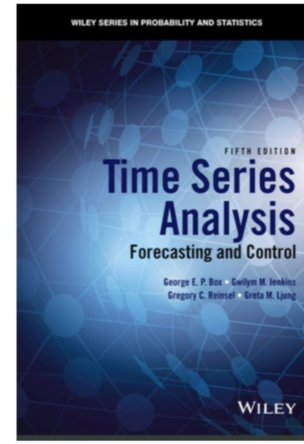
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FIFTH EDITION
Time Series Analysis
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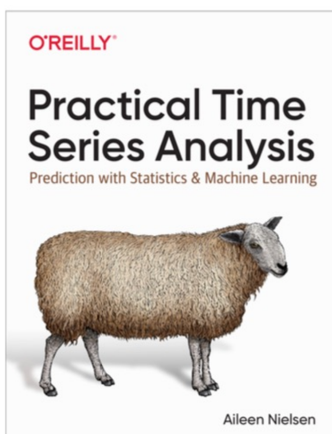
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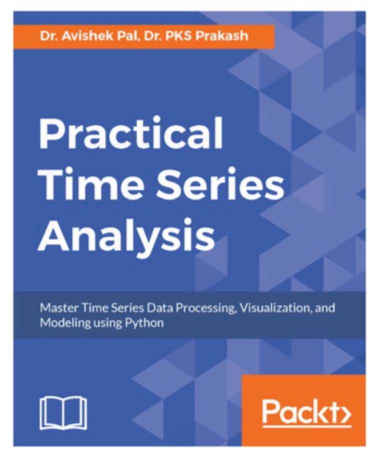
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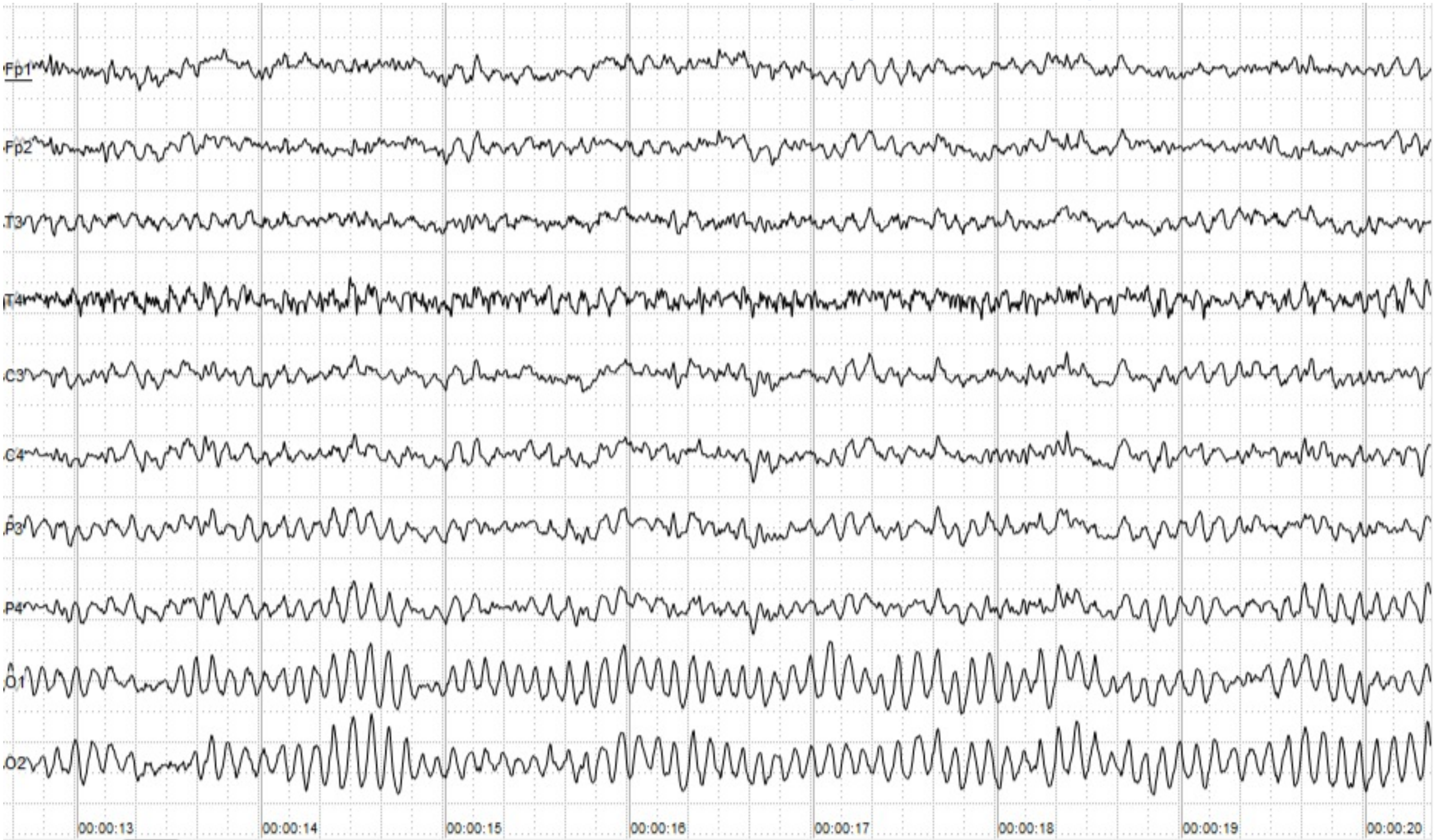

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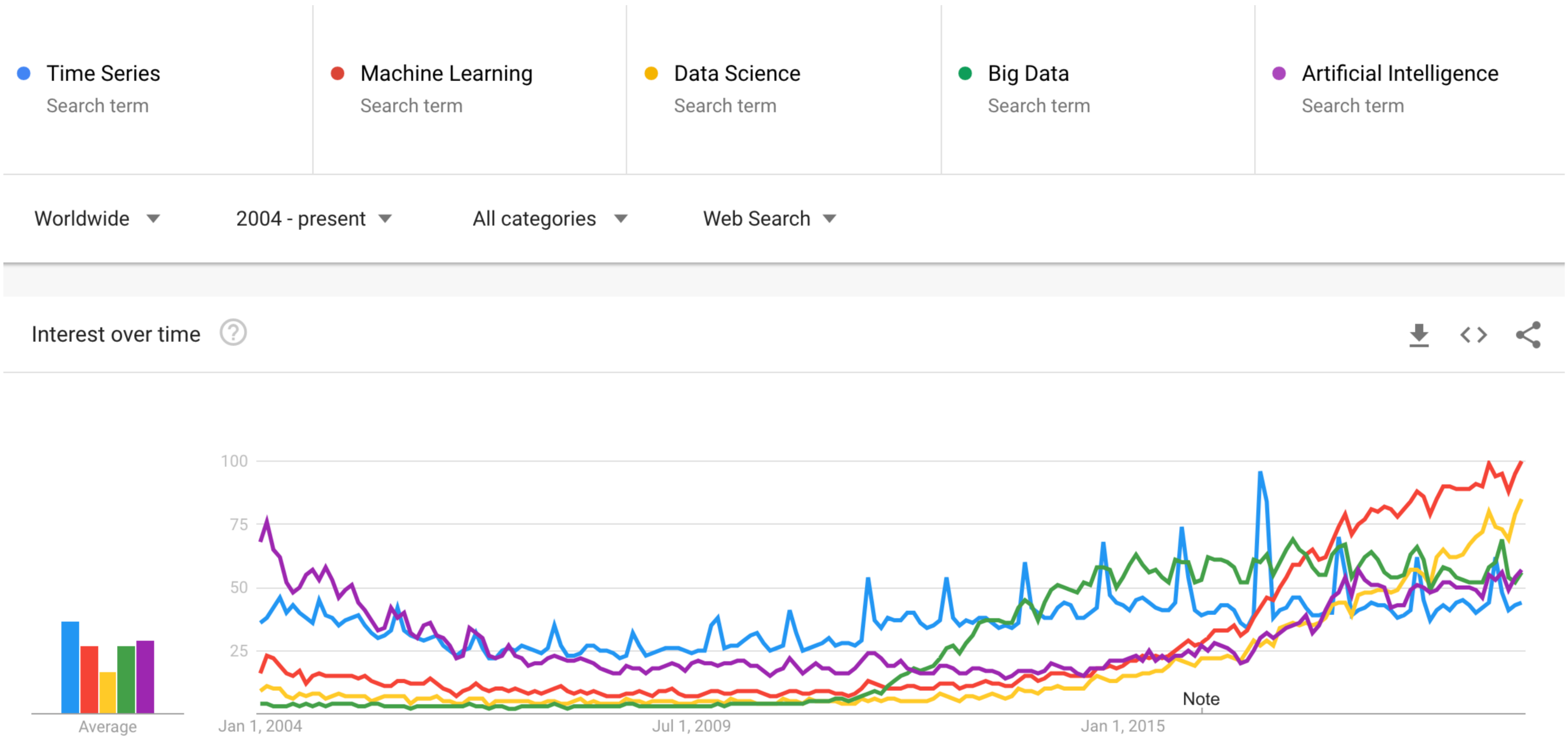
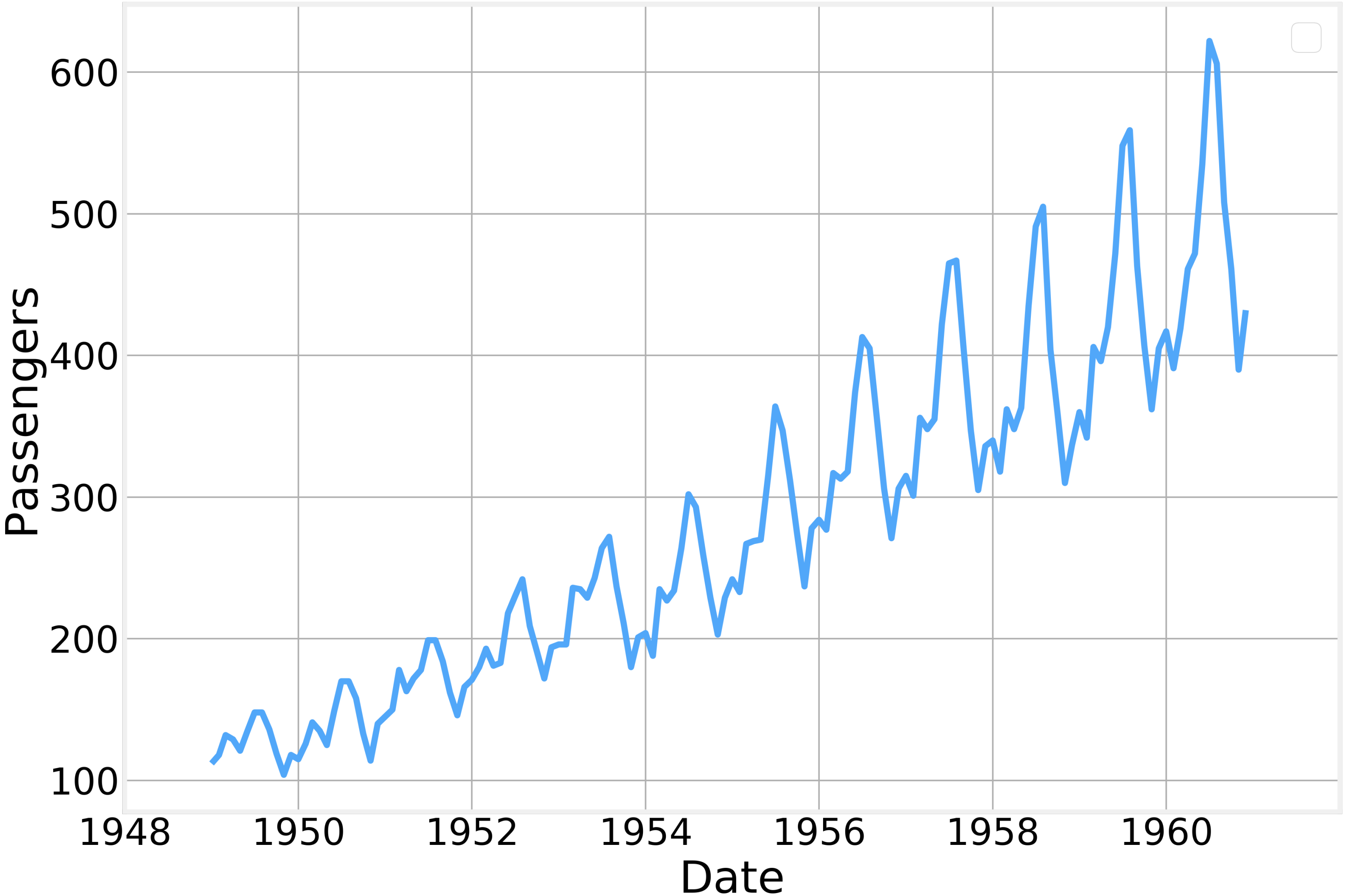
Public Interest – Search Trends



Airline Passengers



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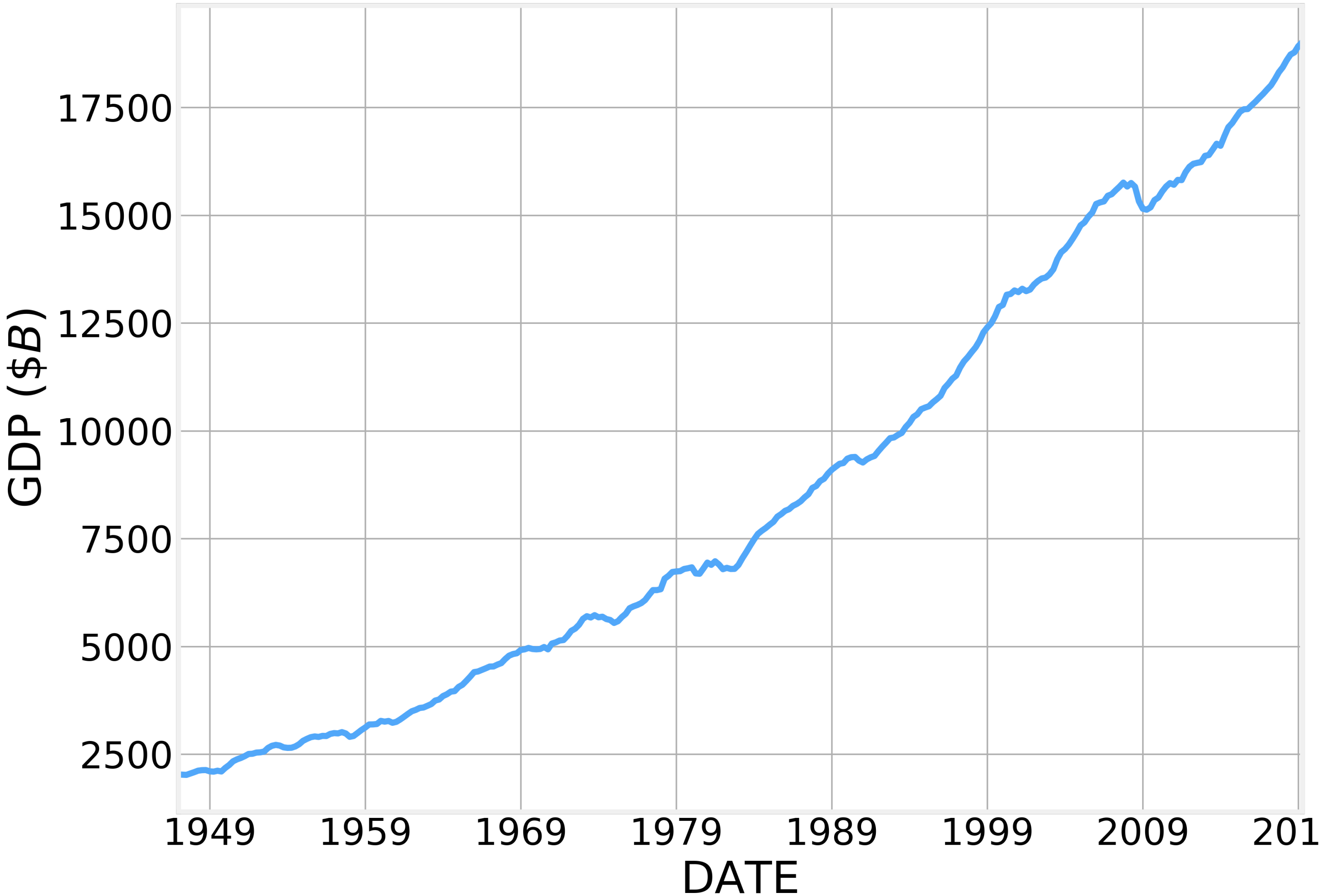

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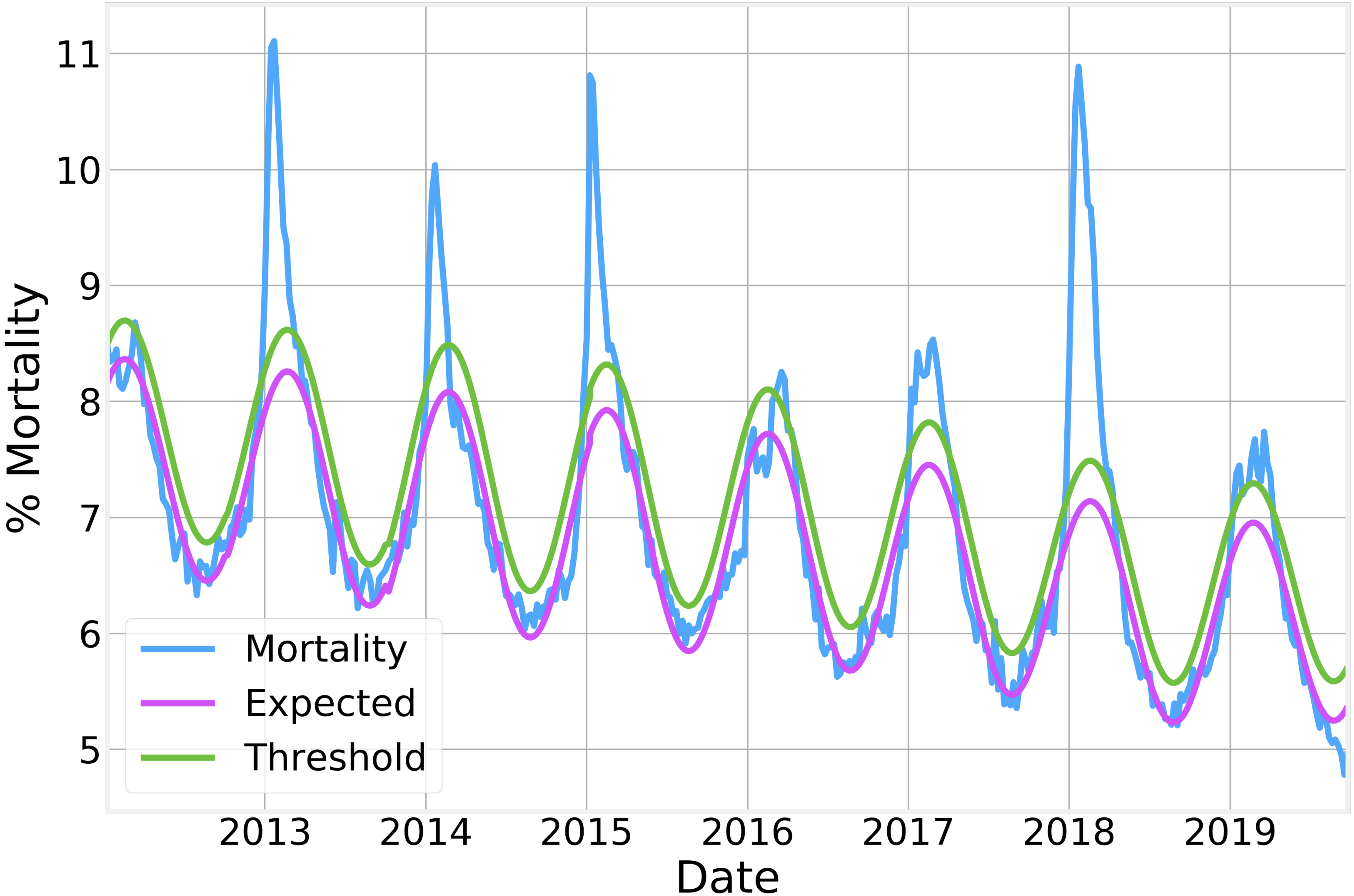

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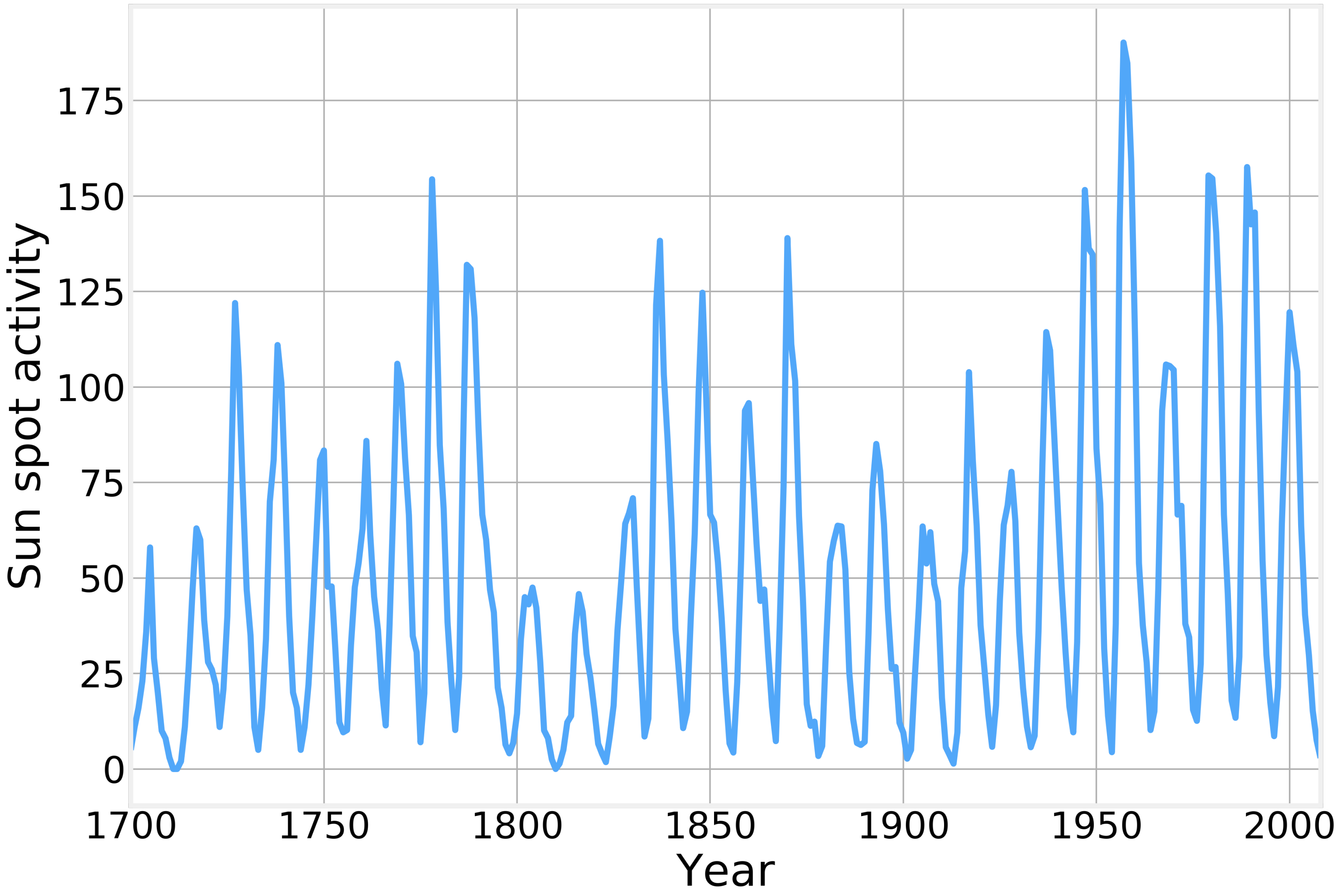

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- Forecasting requires predicting future values based on past behavior

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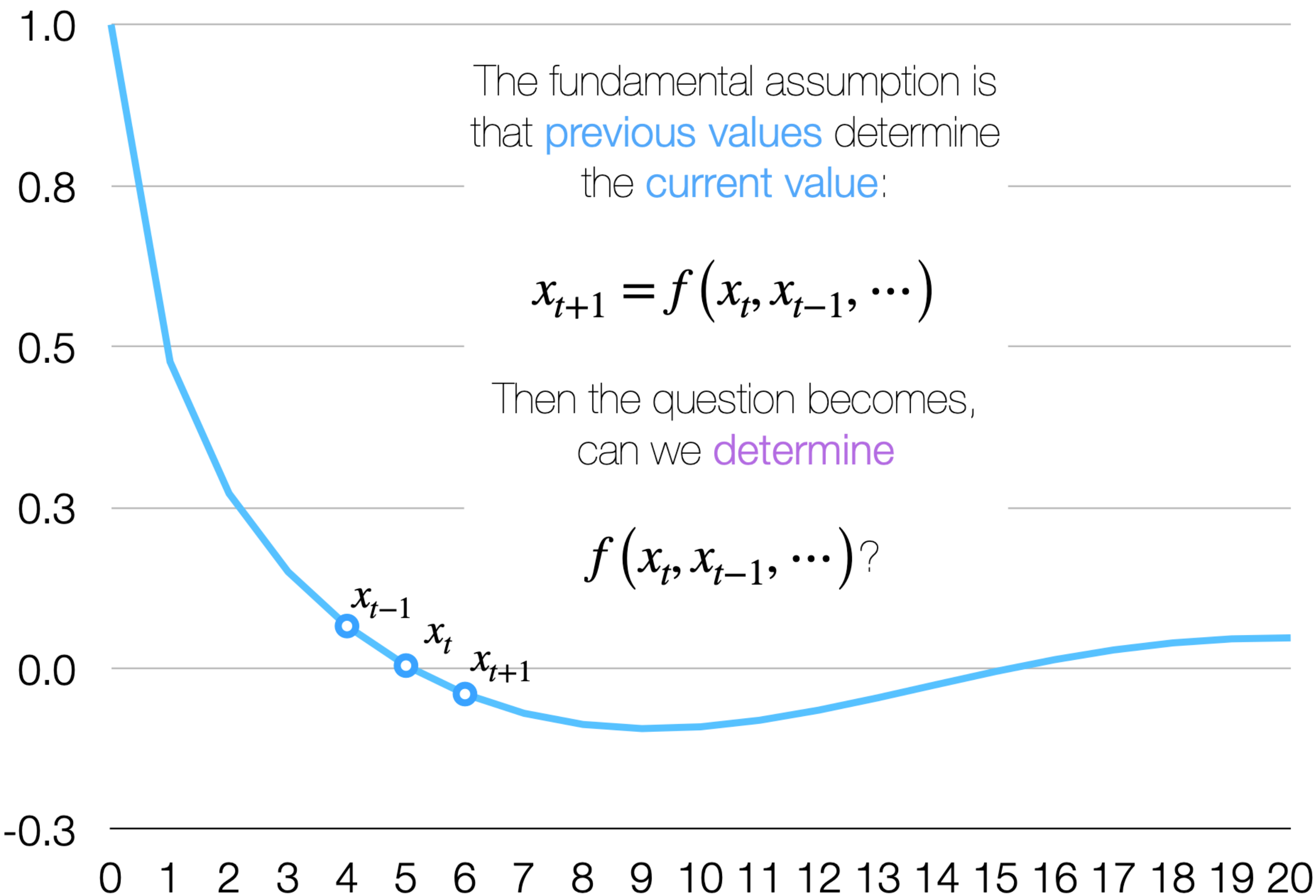
- Running values are calculated over a window of width ***w***

Time Series Analysis



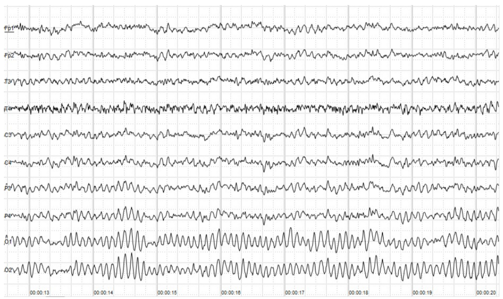
The fundamental assumption is that previous values determine the current value:

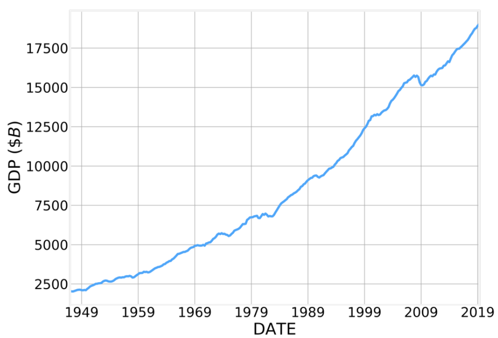
$$x_{t+1} = f\left(x_t, x_{t-1}, \cdots\right)$$

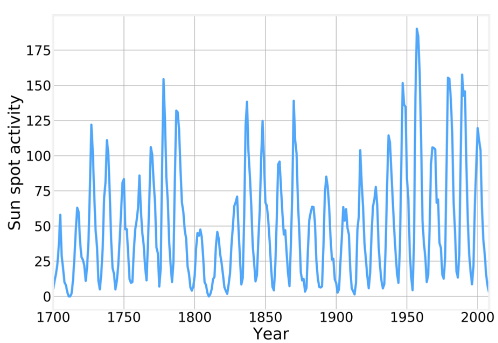
Then the question becomes, can we determine

$$f\left(x_t, x_{t-1}, \cdots\right)?$$

Three fundamental behaviors







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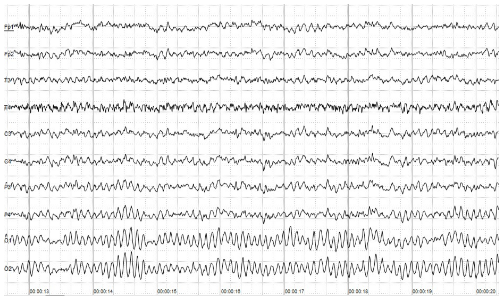
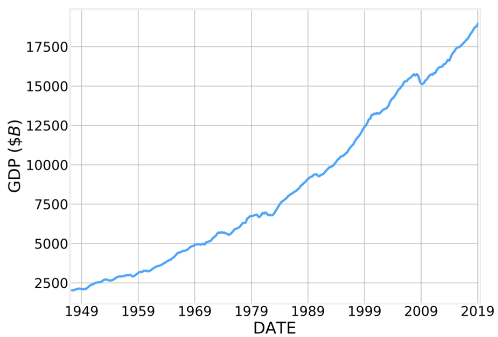


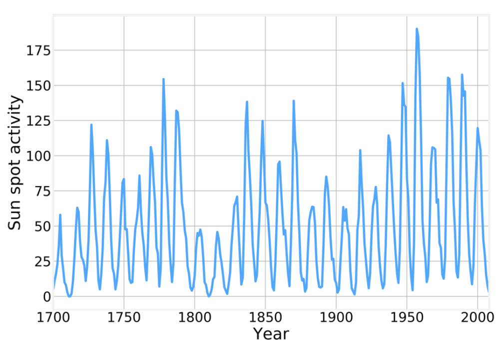
Stationarity

$\langle x_t \rangle \approx \text{constant}$



Trend

$\langle x_t \rangle \approx ct$



Seasonality

$X_{t+T} \approx x_t$

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- Typically, the first step of any analysis is to transform the series to make it stationary

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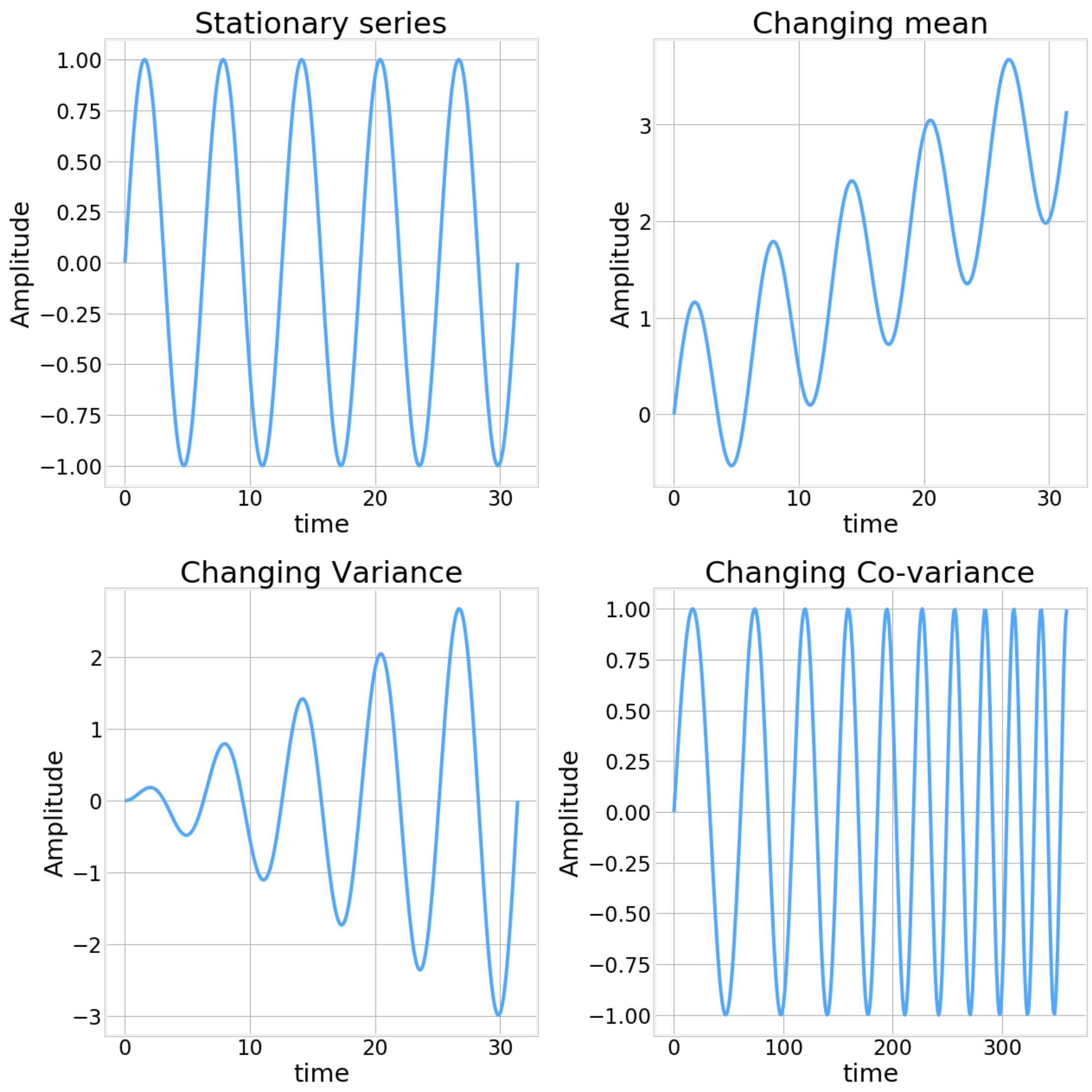

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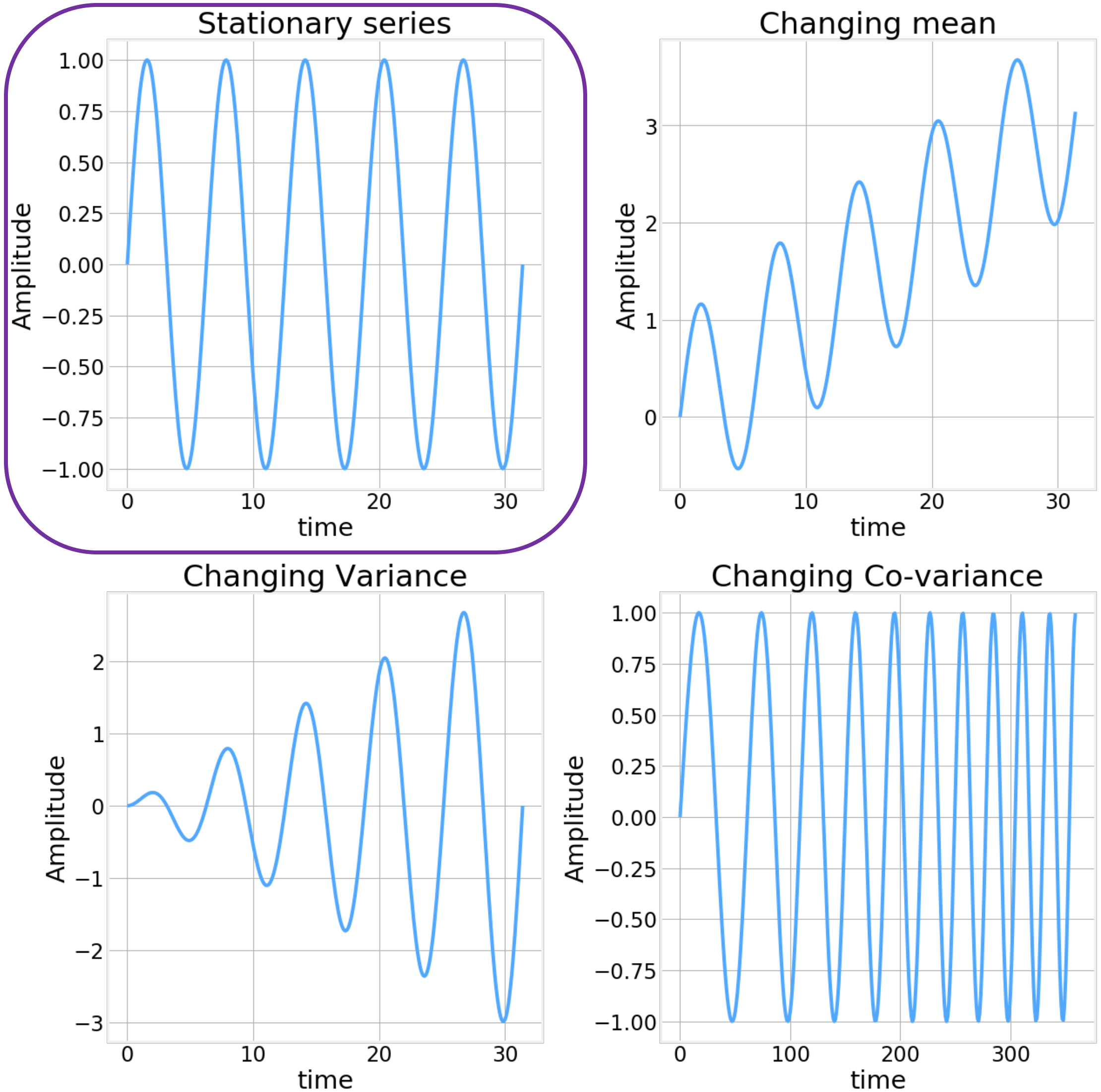

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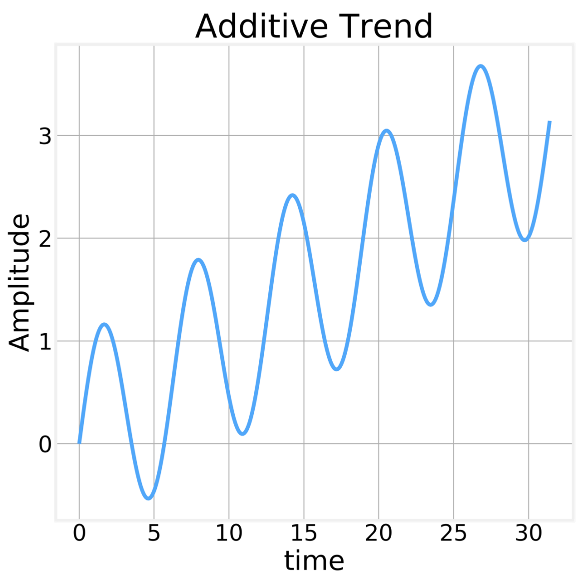

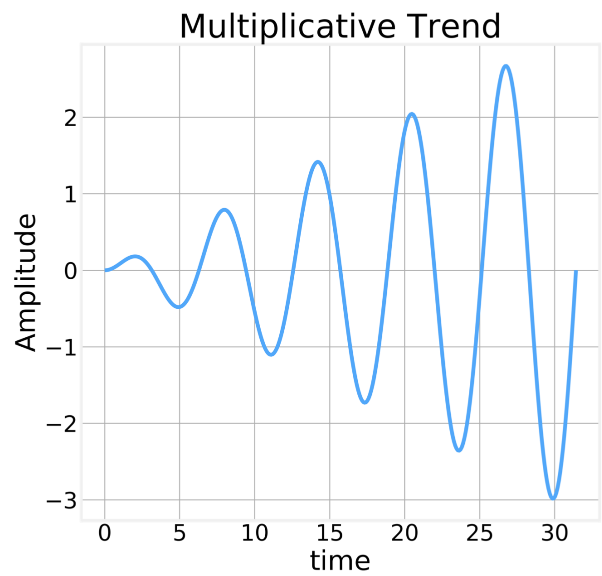

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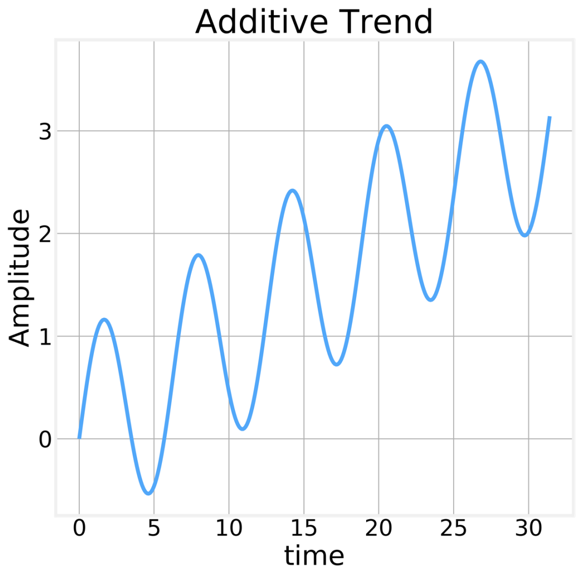

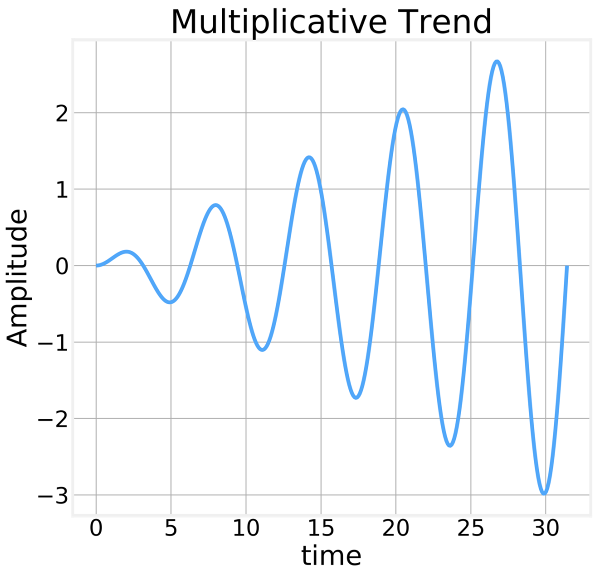

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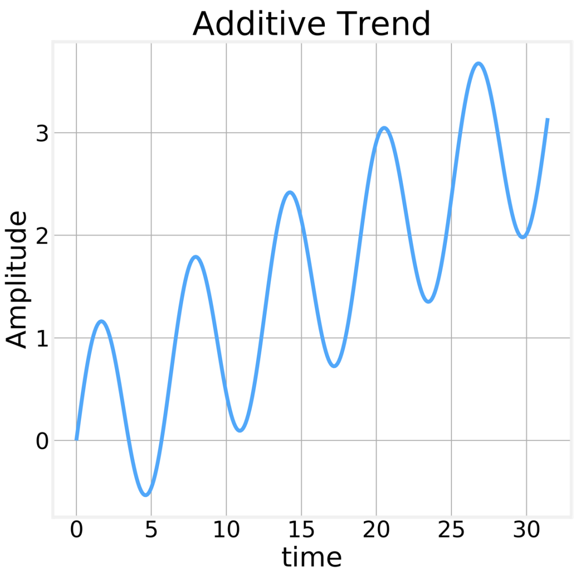

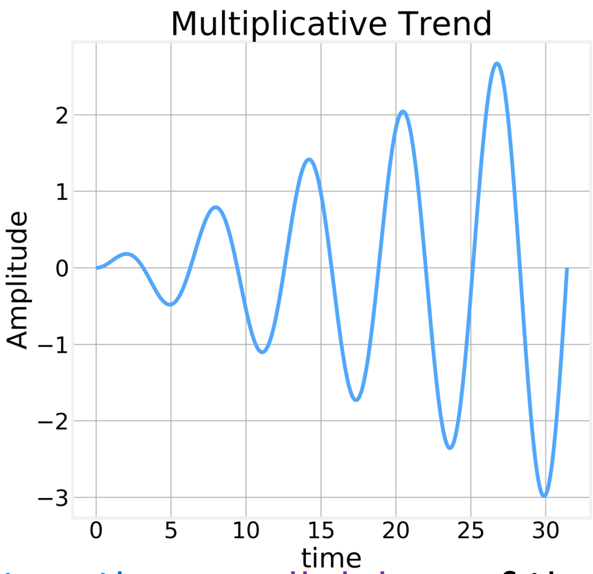

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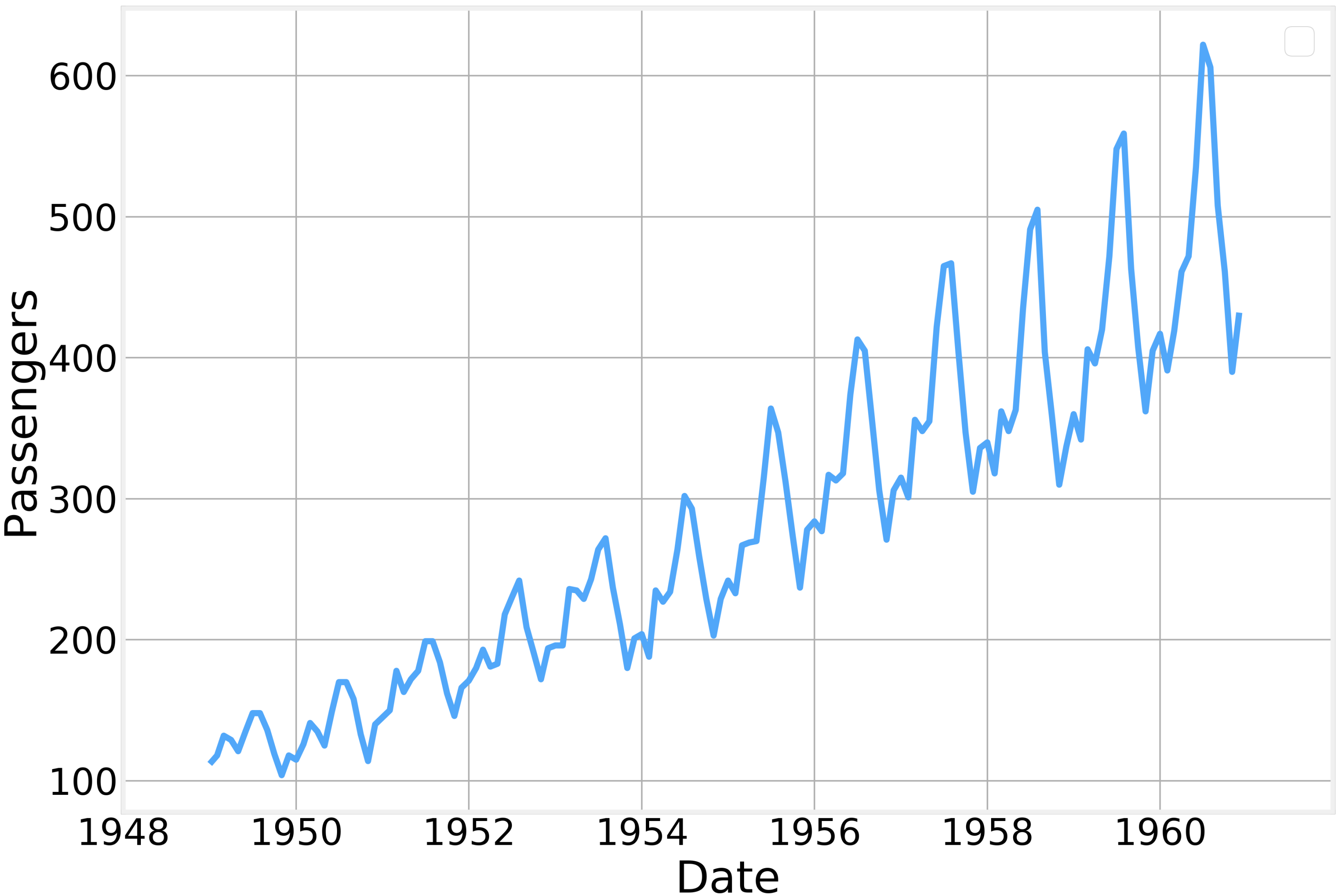
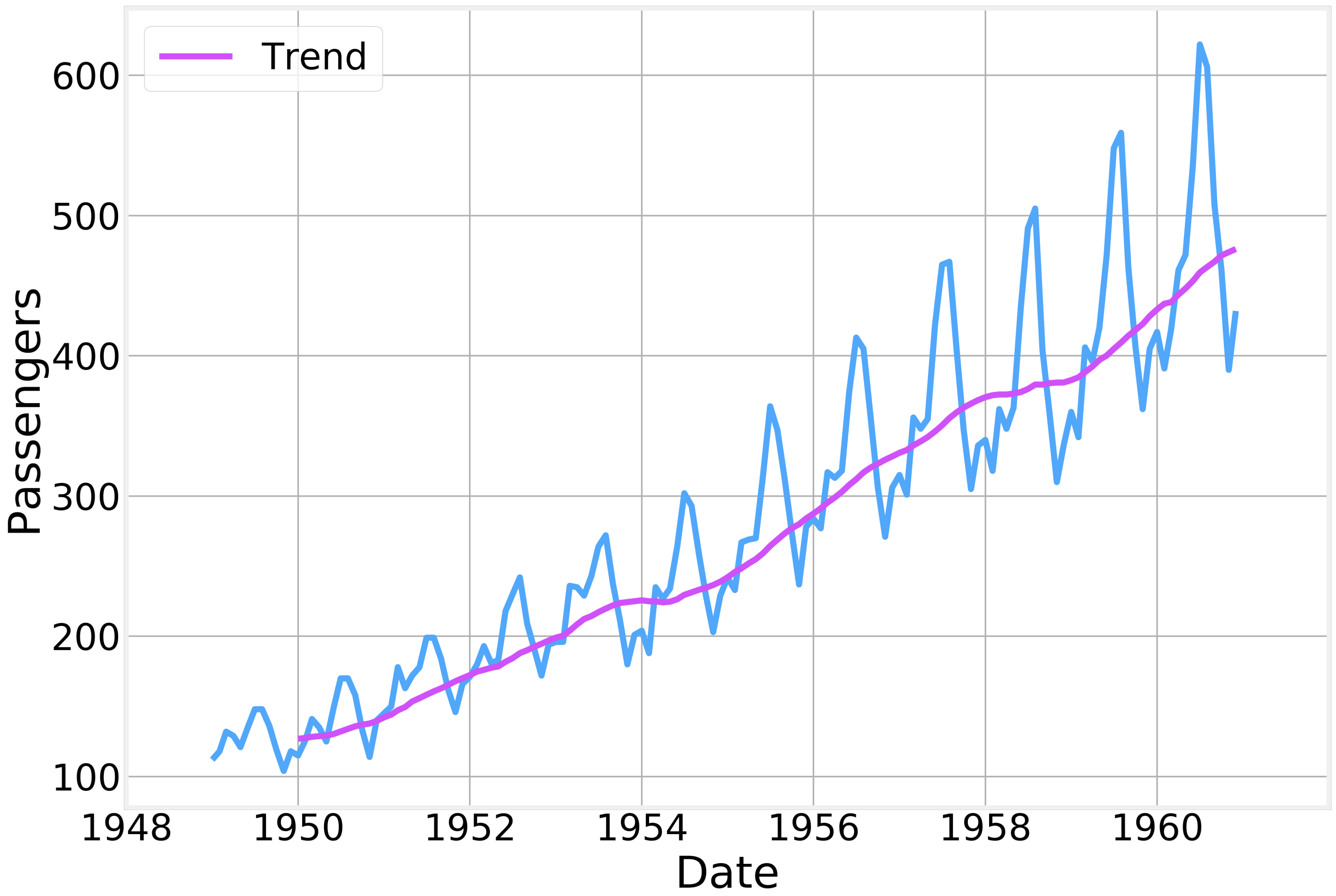
- One way to determine the trend is to find the running average

Trend



600
500
400
300
200
100
Passengers
1948
1950
1952
1954
1956
1958
1960
Date

Trend



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- Understanding the seasonality of a time series provides important information about its long-term behavior and is extremely useful in predicting future values

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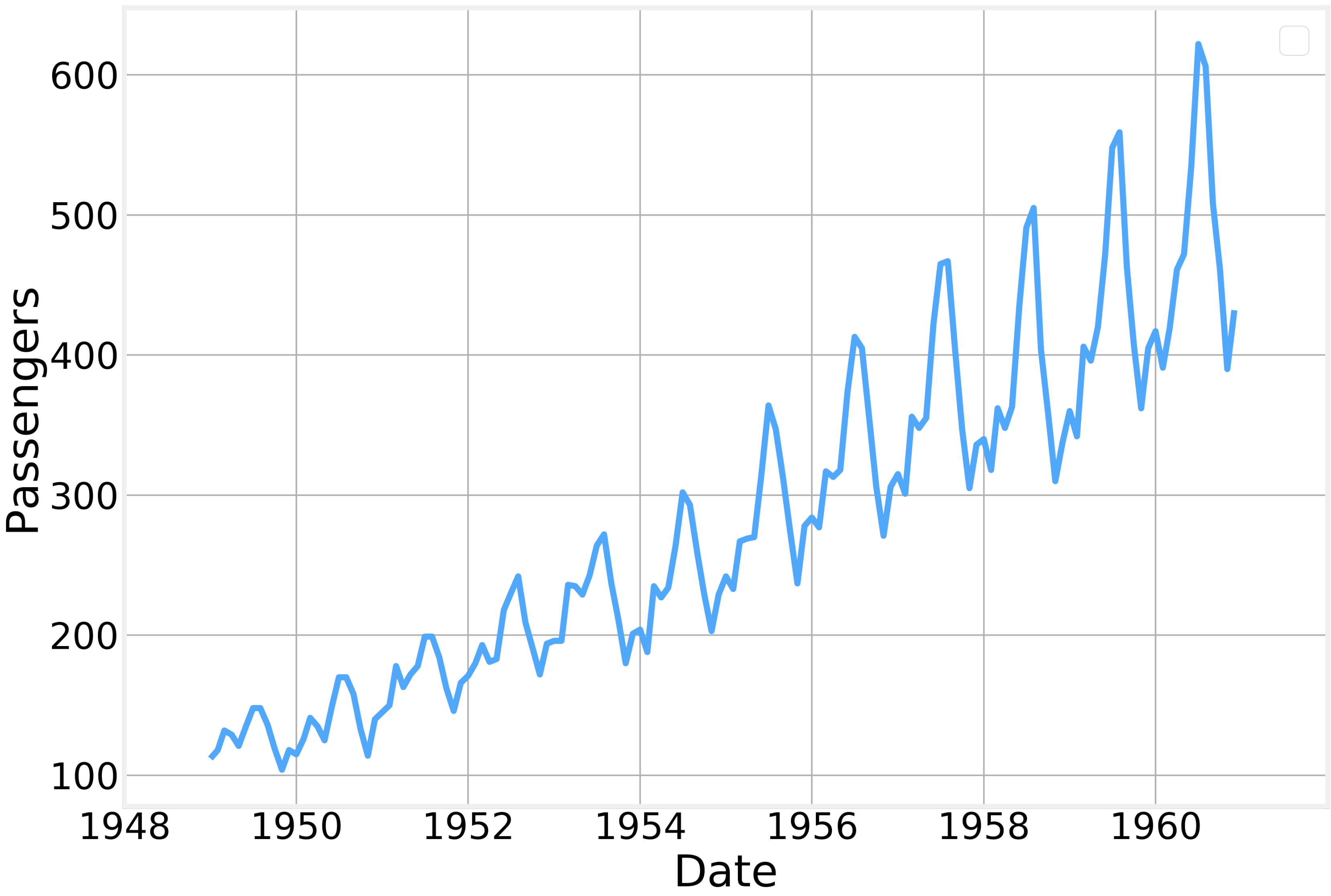

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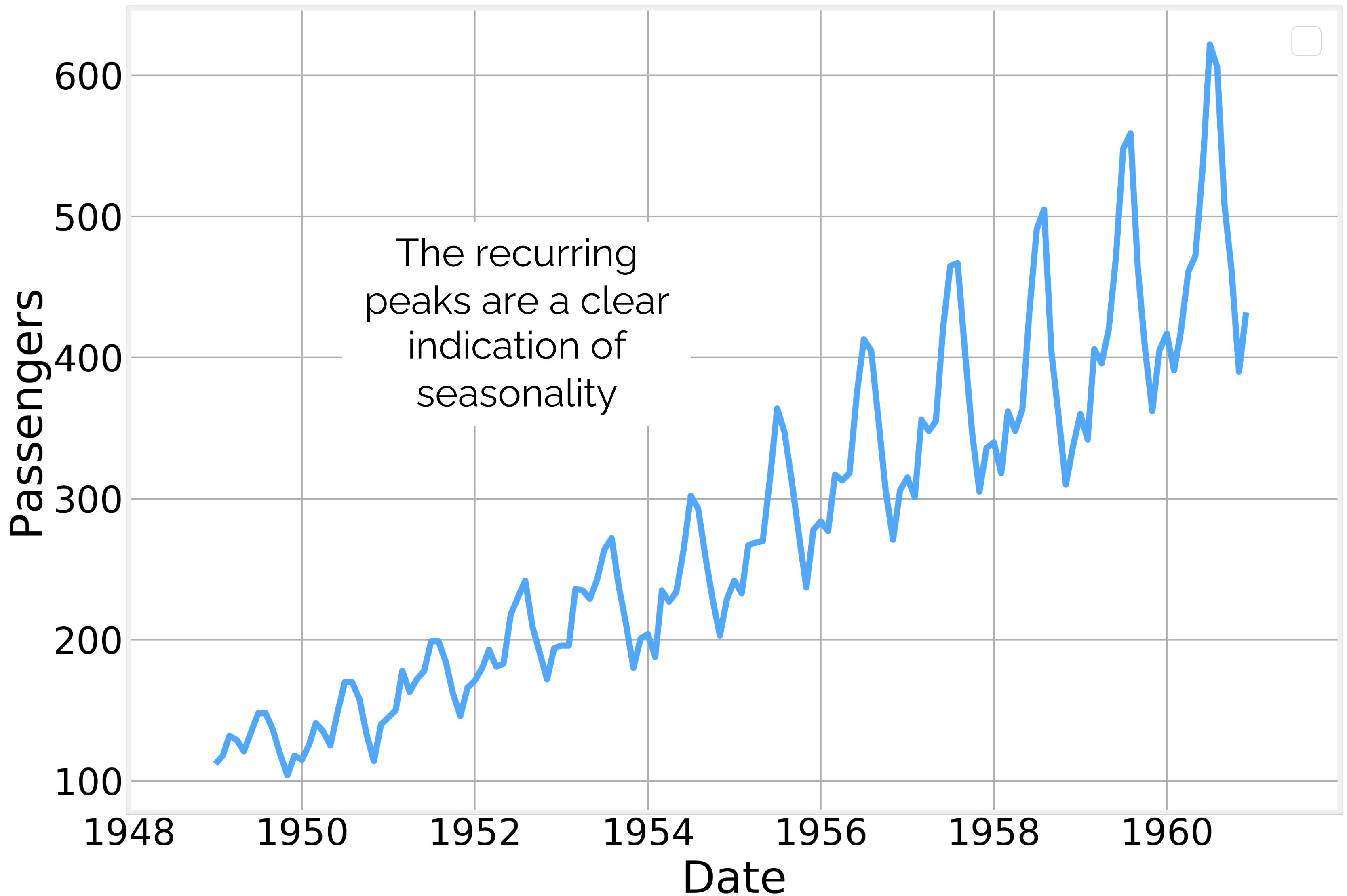

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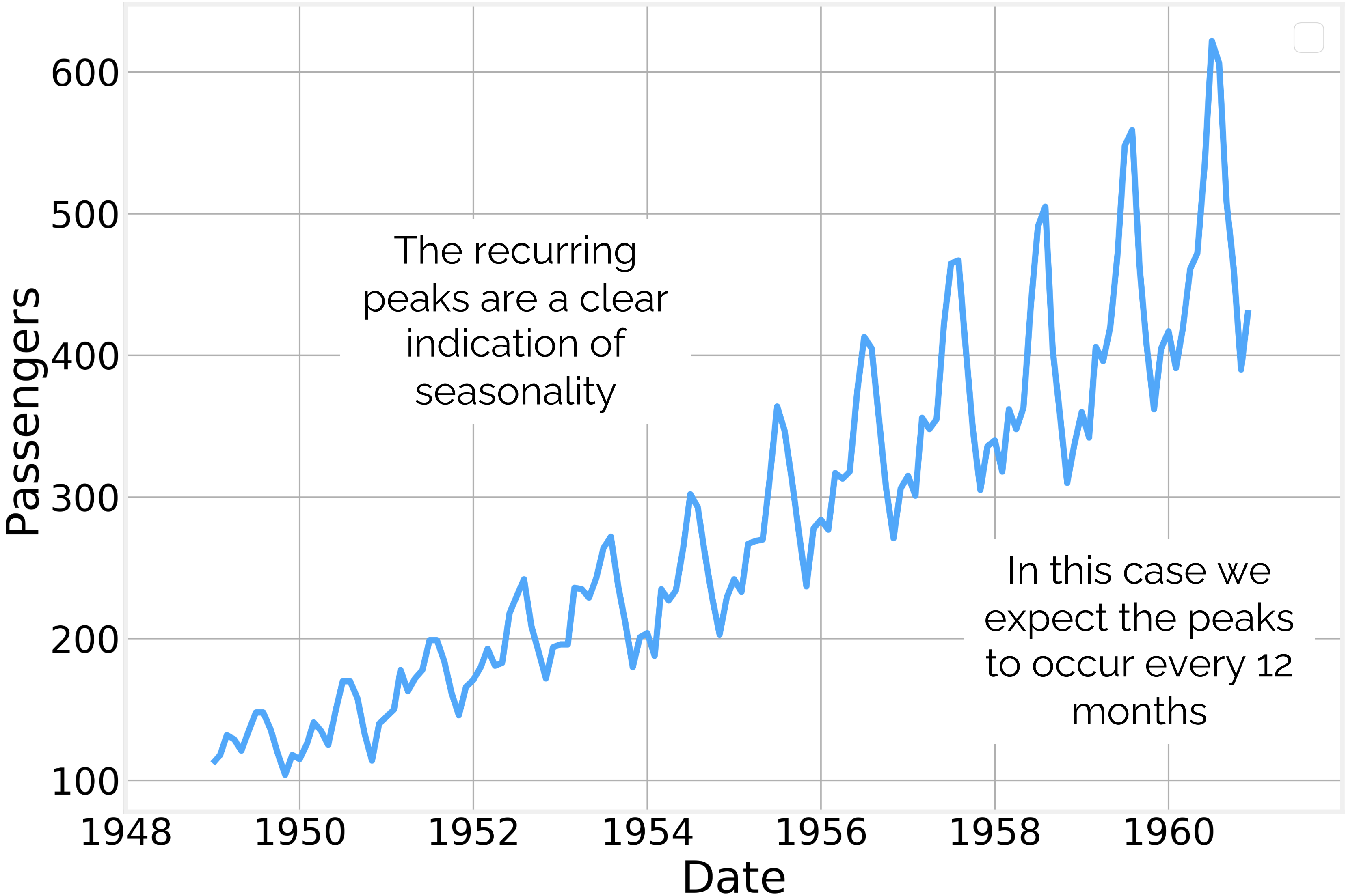

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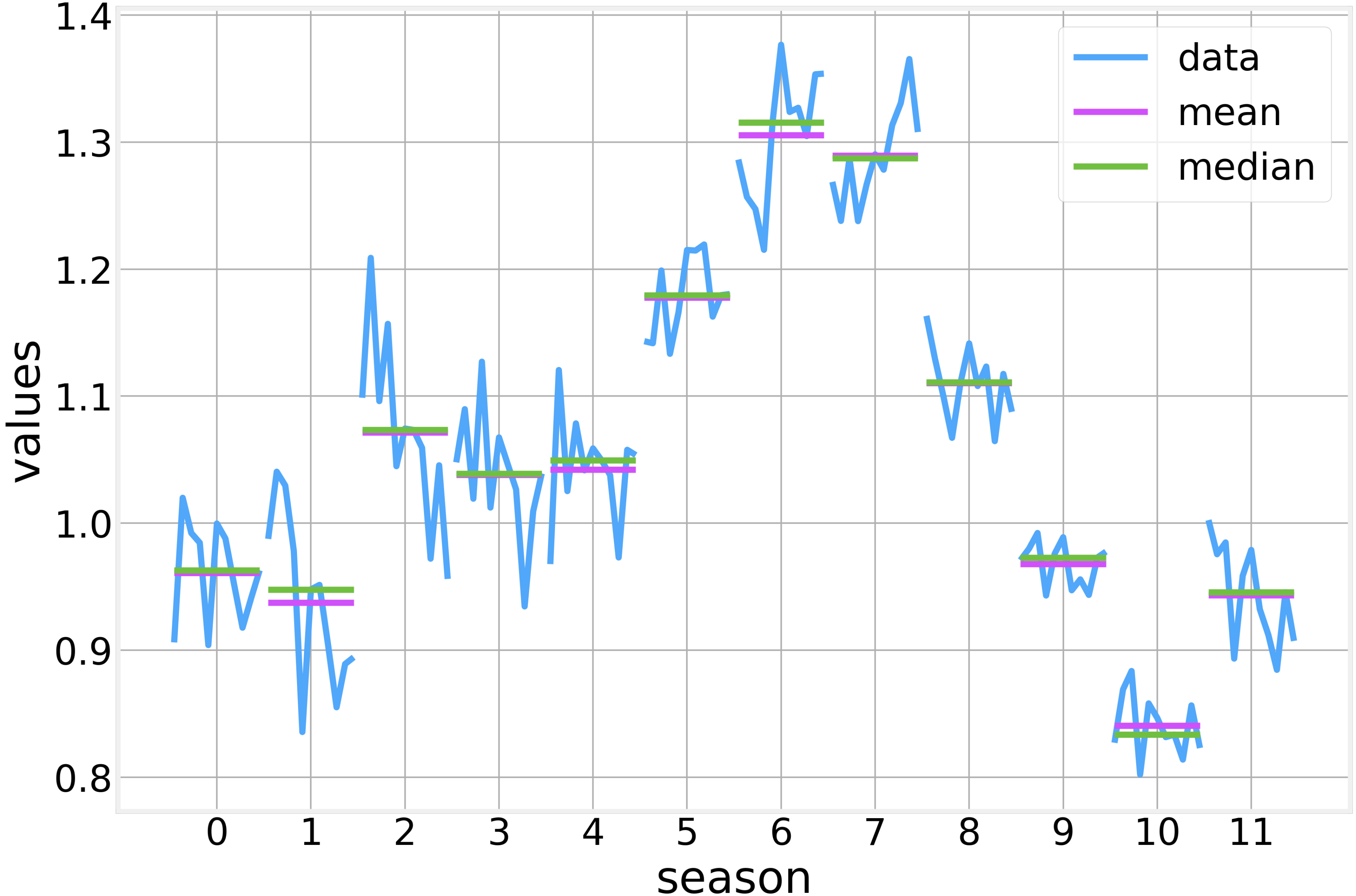

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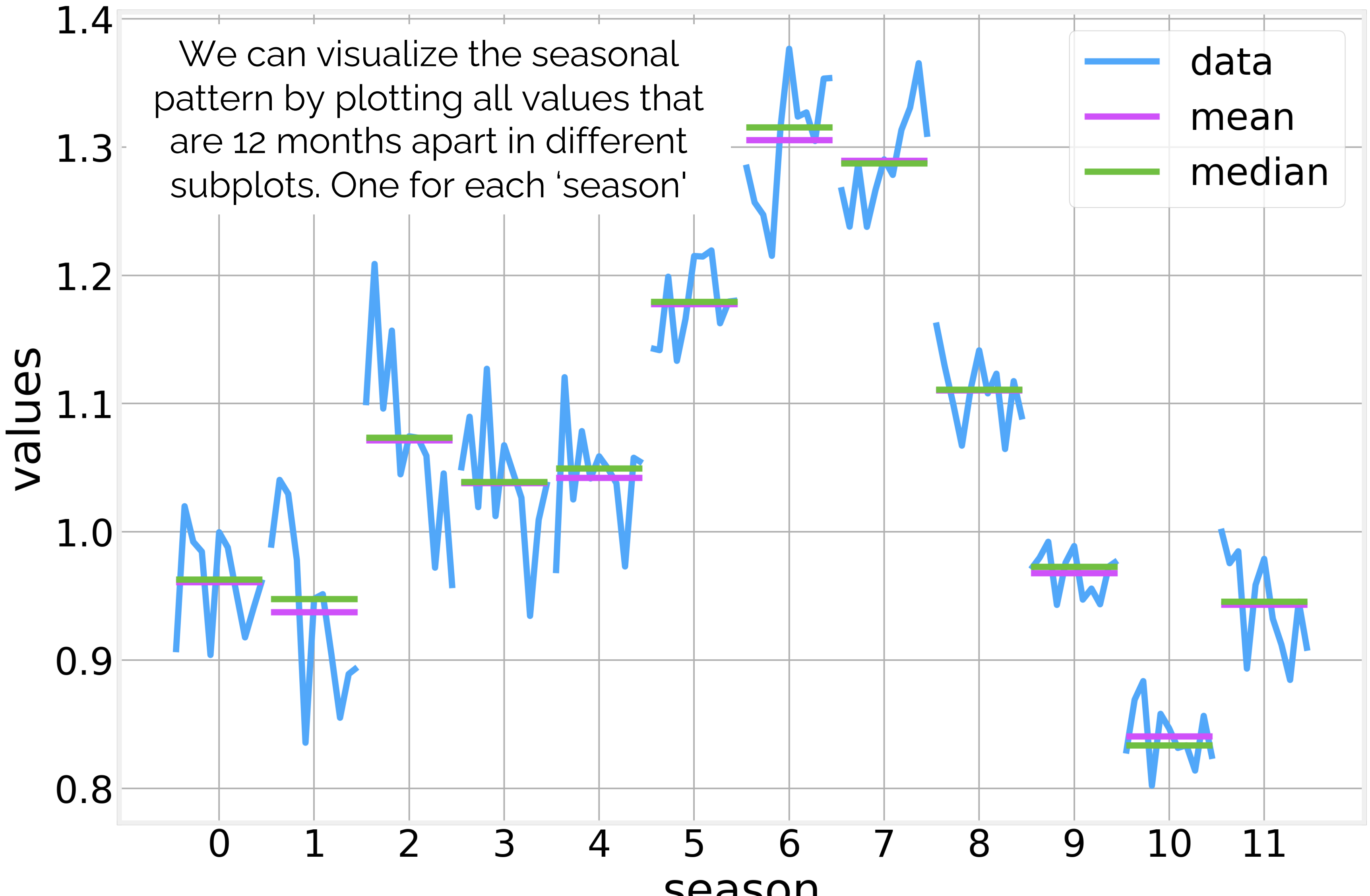

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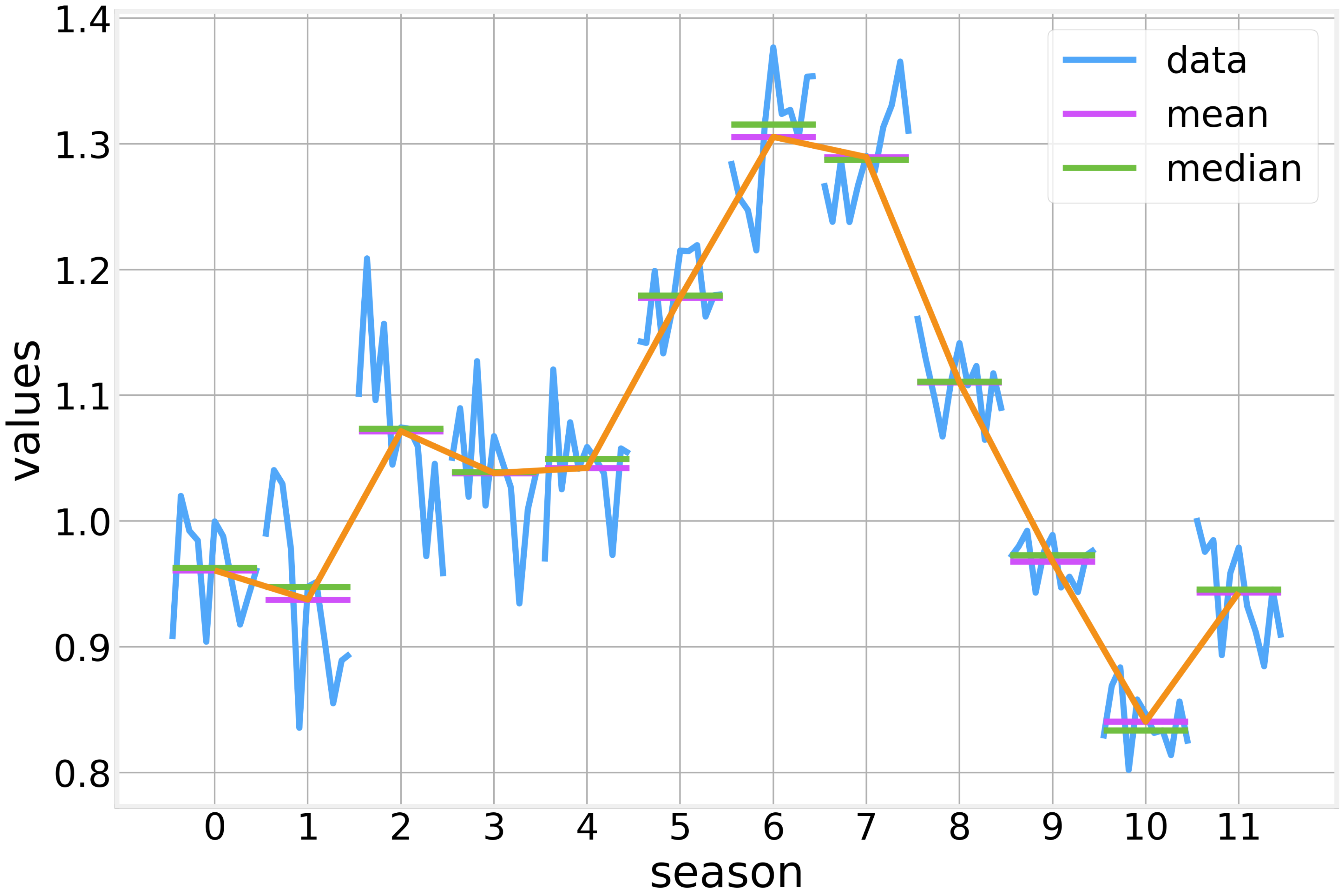

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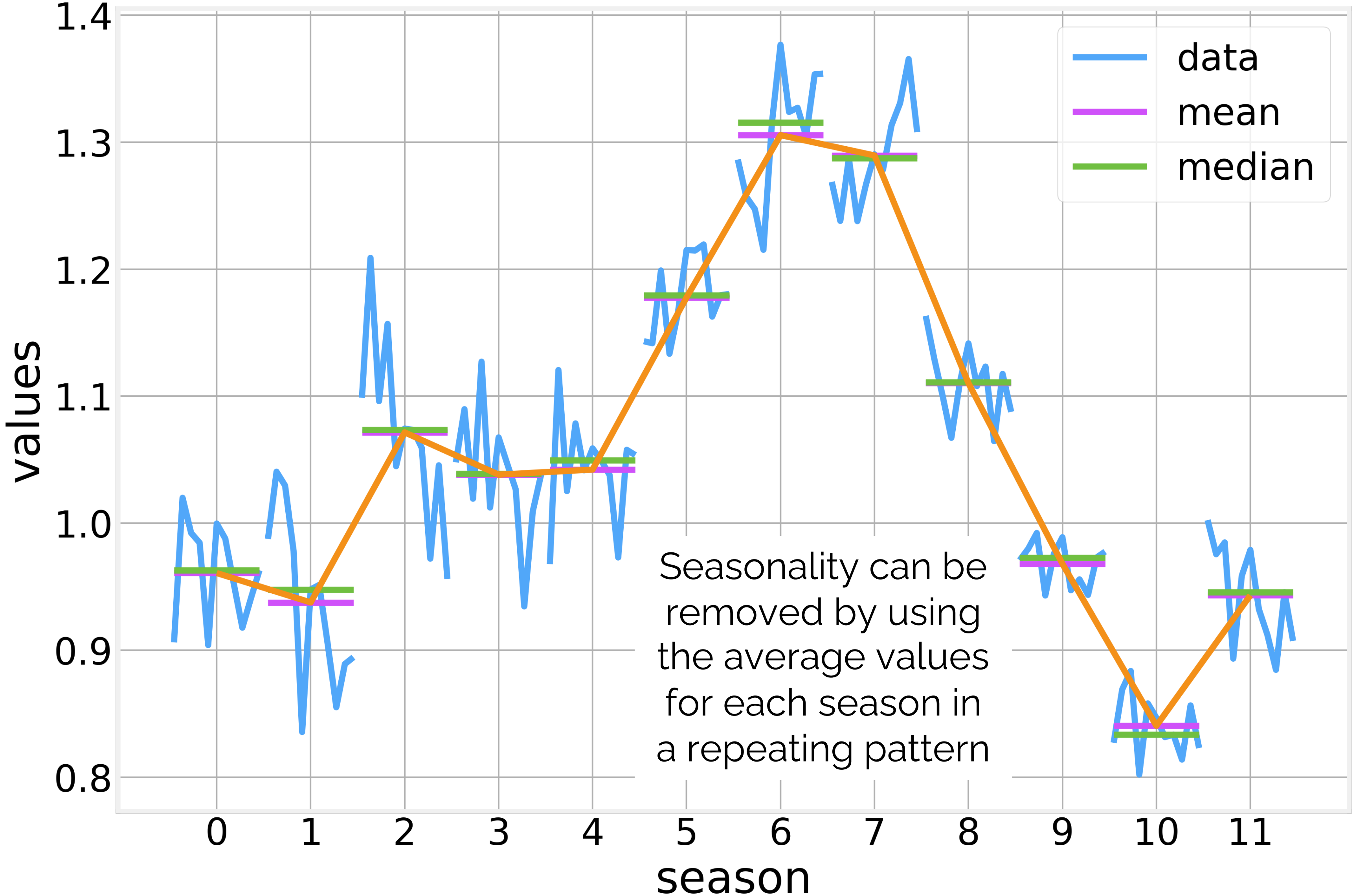

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- Residuals are typically stationary

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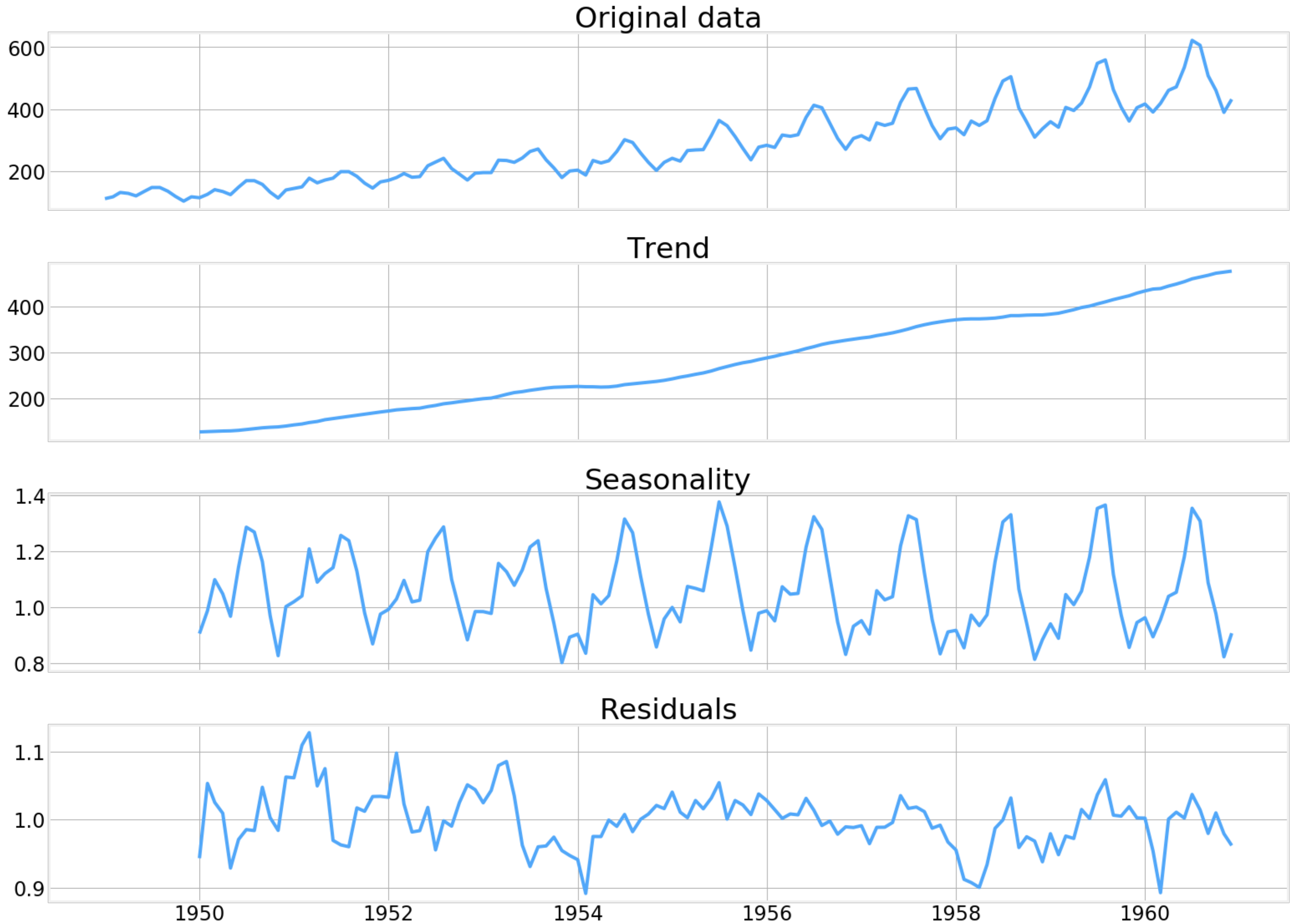


Lesson 2:

Processing Timeseries Data

Lagging Values

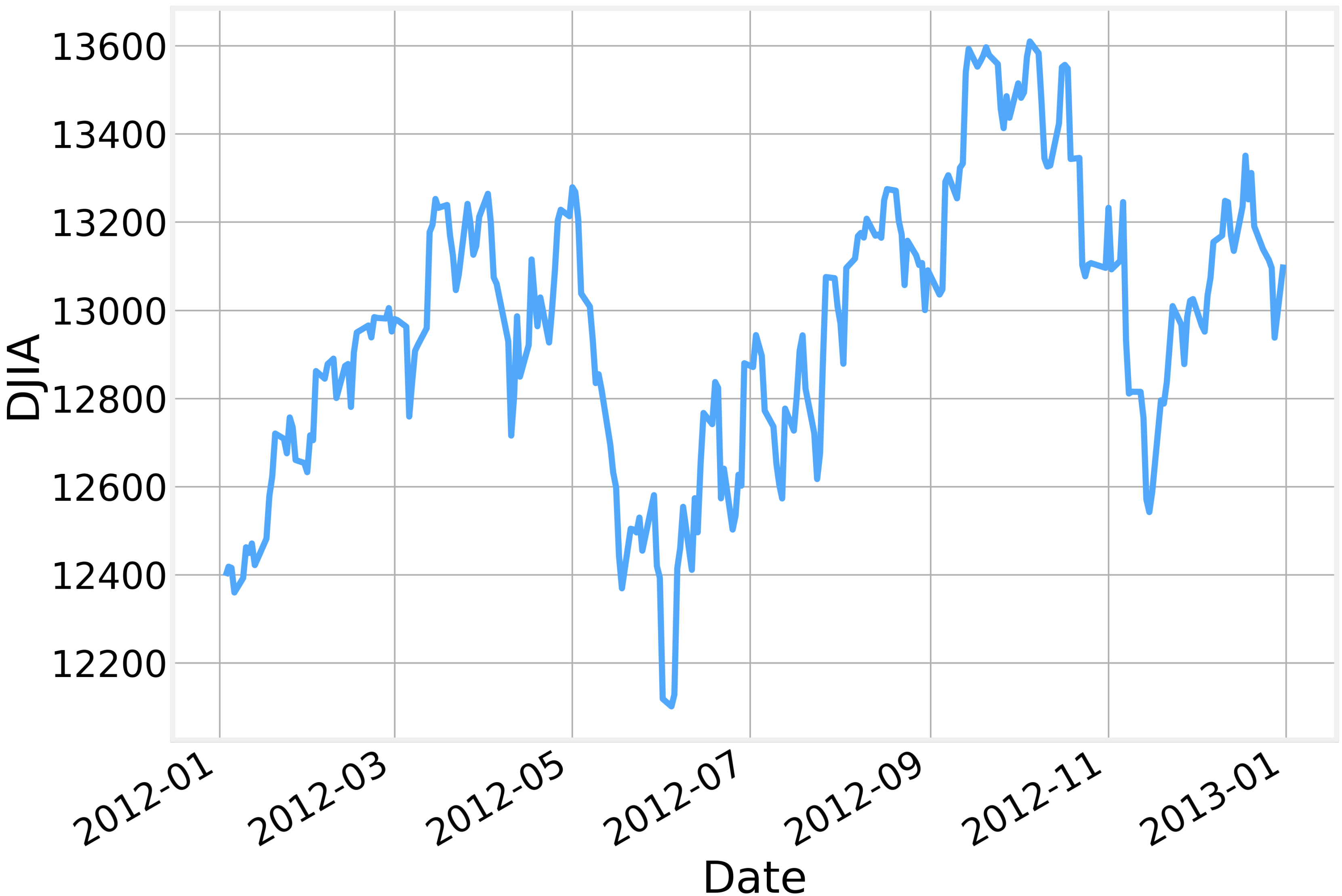
- While analyzing time series, we often refer to values that our time series took **1**, **2**, **3**, etc., time steps in the past

- These are known as lagged values and denoted:

$$X_{t-l}$$

- where l is the value of the lag we are considering.

Lagging Values



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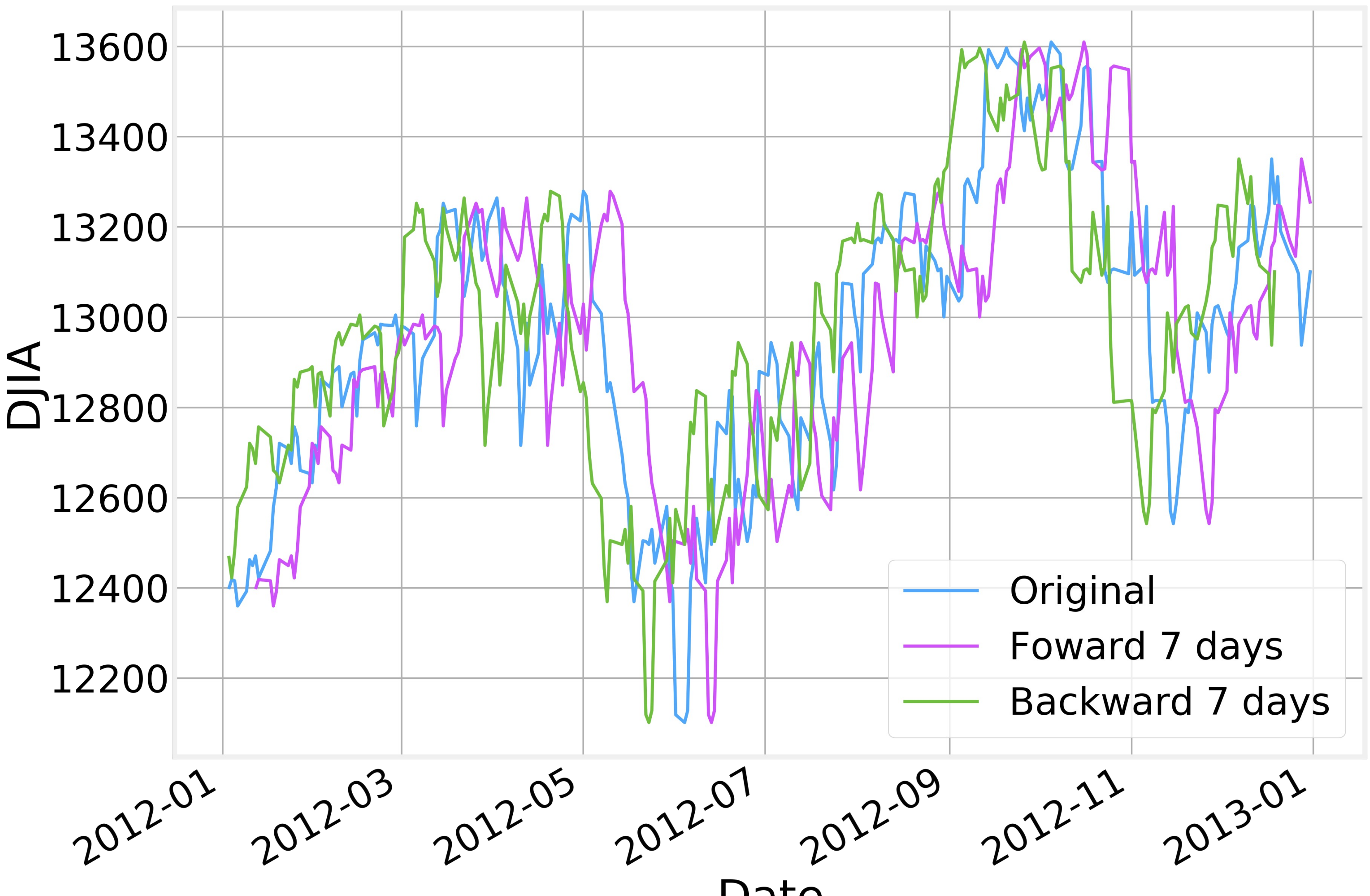


Differences

- Perhaps the most common use case for lagged values is for the calculation of differences of the form:

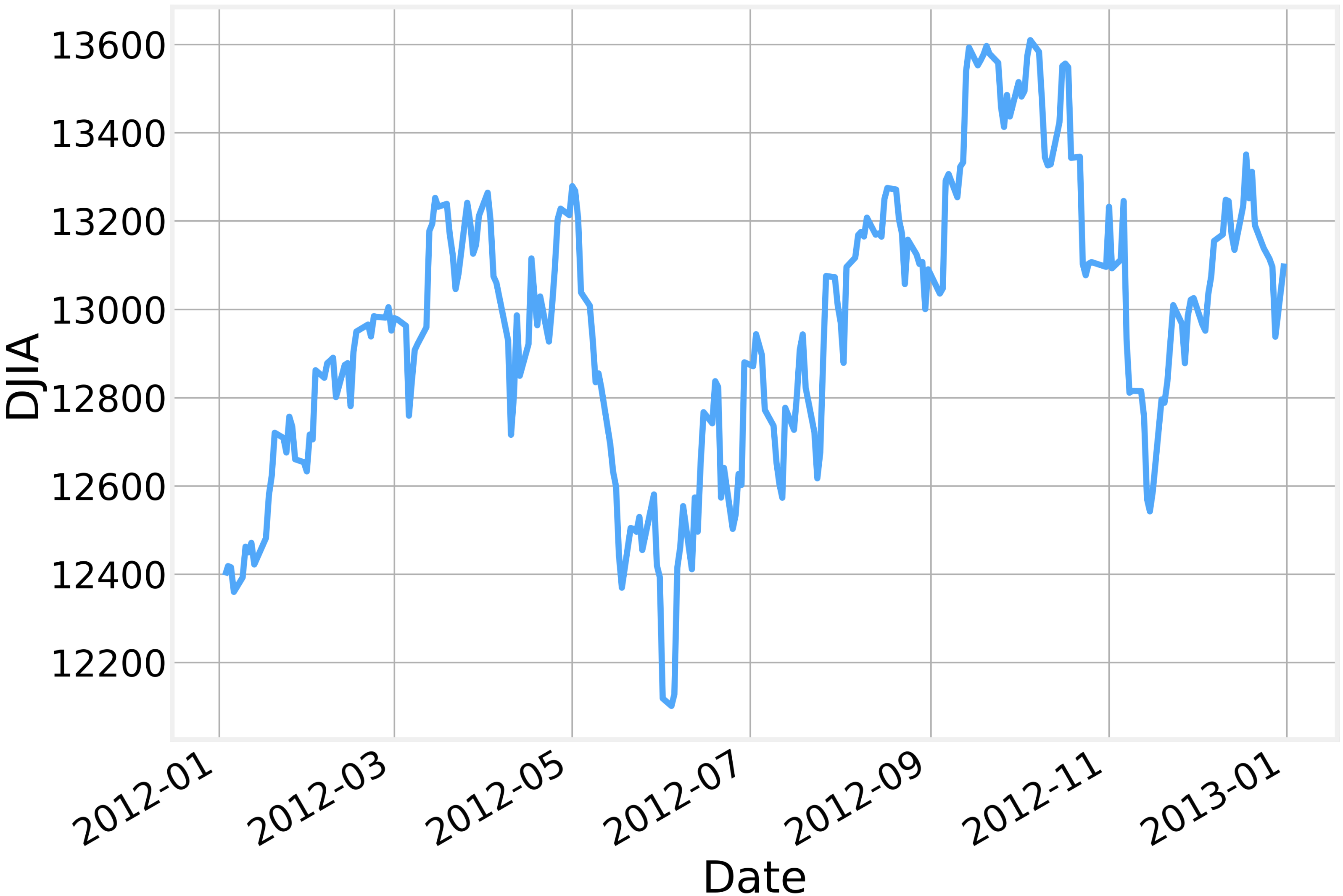
$$X_t - X_{t-l}$$

- Where $l \geq 1$ is the value of the lag we are interested in.
- Naturally, higher order differences can also be used, in which case, the difference of the difference is calculated:

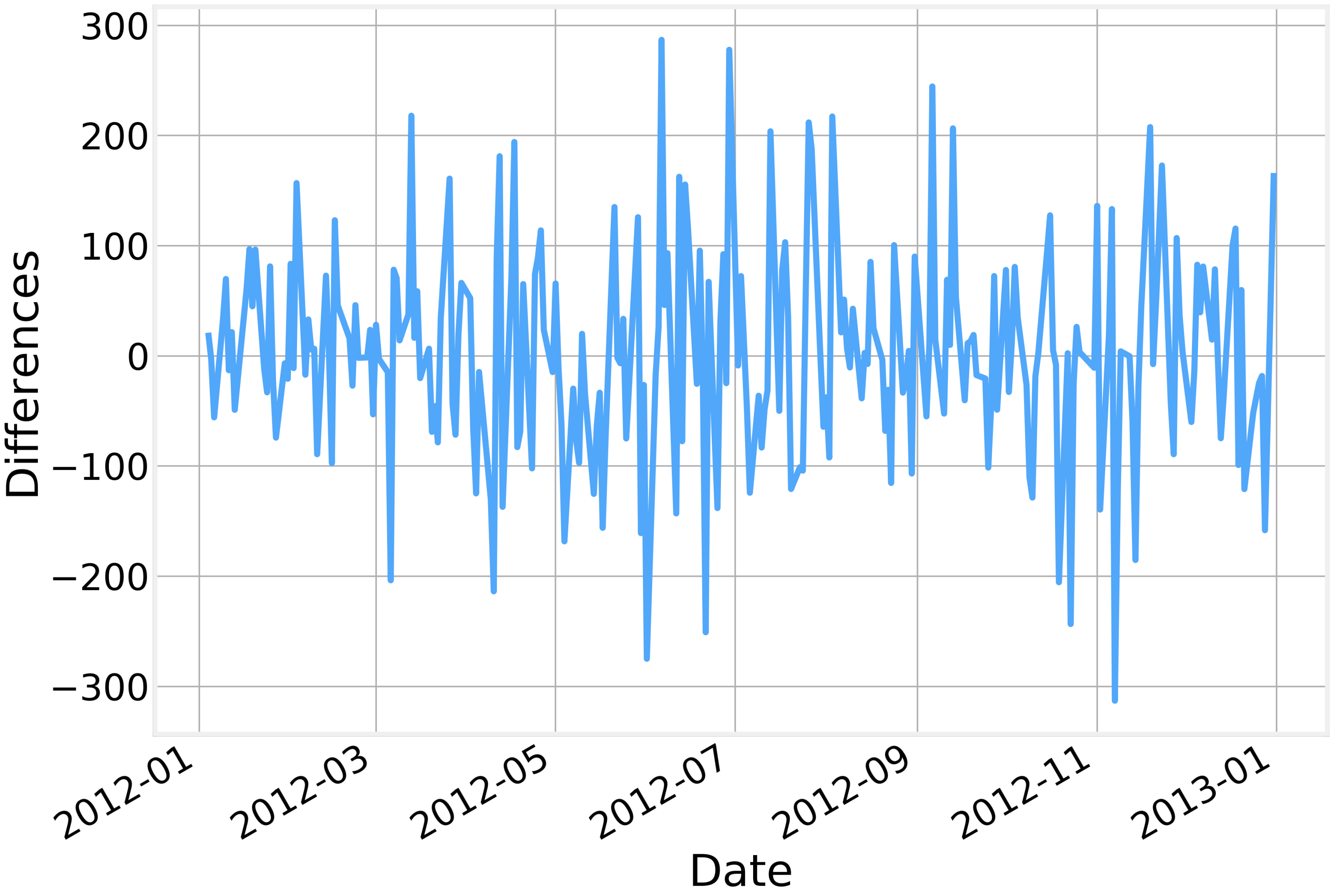
$$y_t = x_t - x_{t-l}$$
$$z_t = y_t - y_{t-l} \equiv x_t - 2x_{t-l} + x_t - 2l$$

- This can be thought of as a discrete version of the usual derivative of a function.
- Differences are also a particularly simple way to detrend a time series

Differences



Differences



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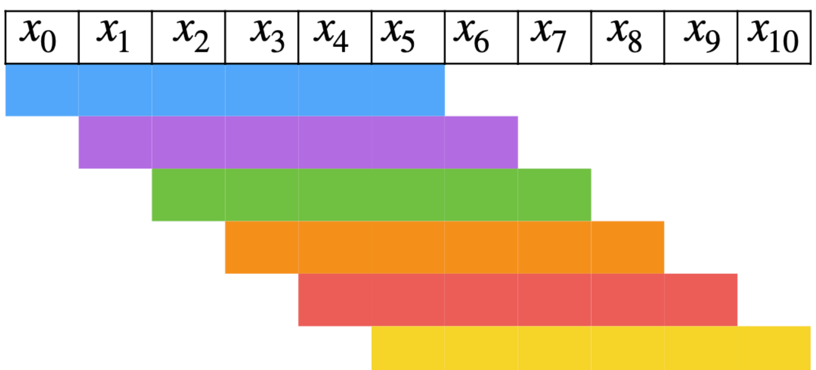
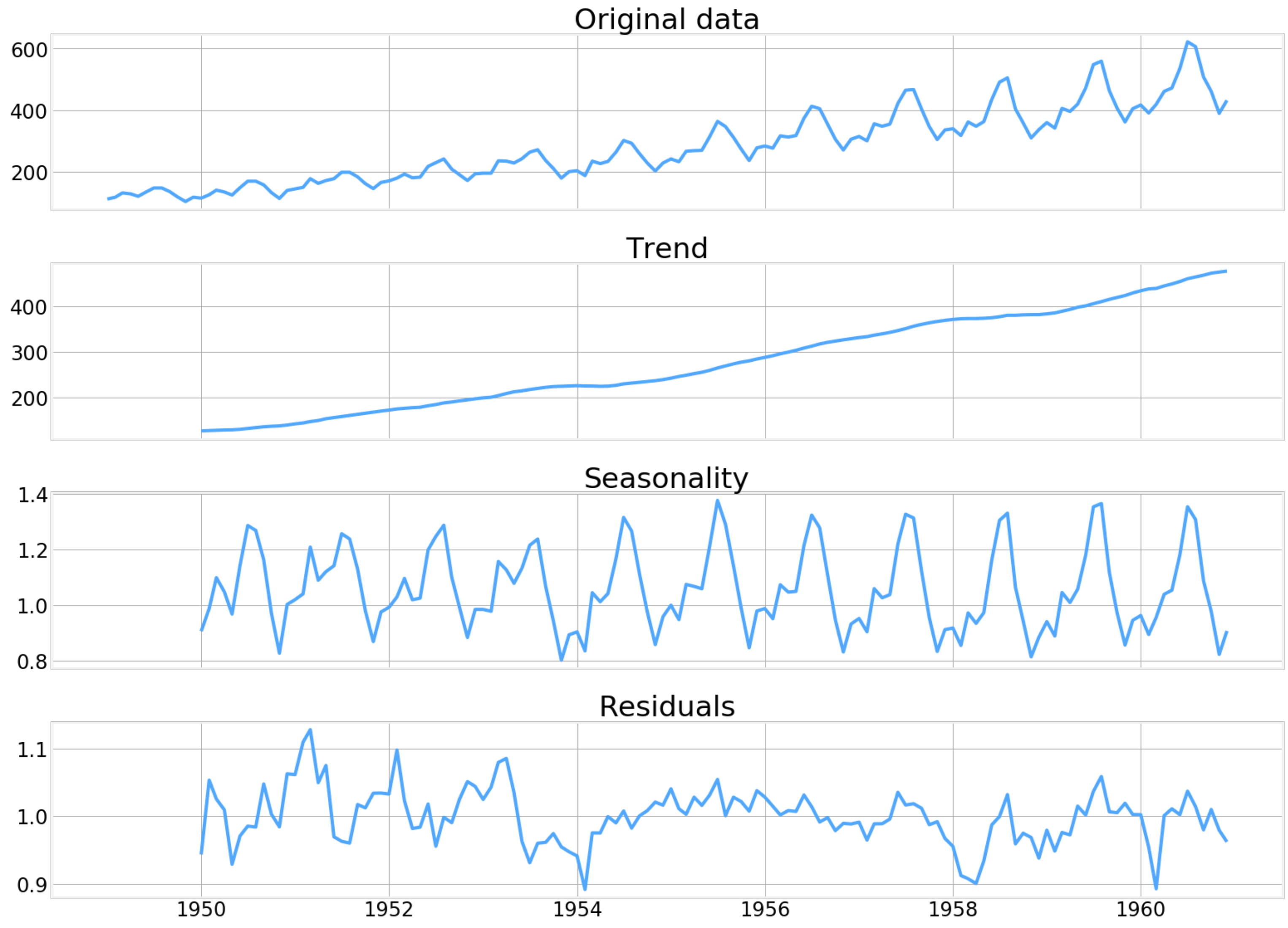


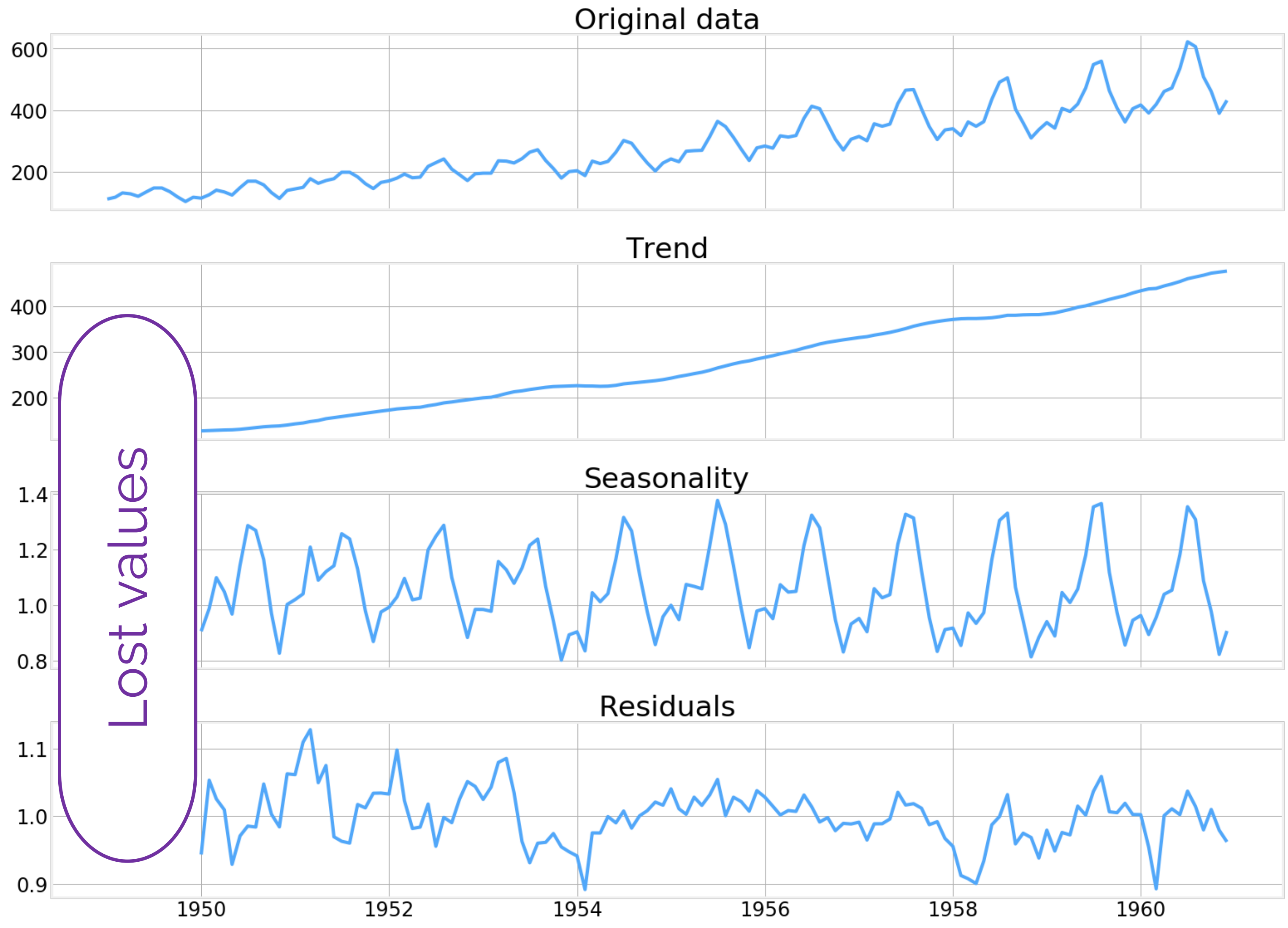
$$= \begin{matrix} x_0 & x_1 & x_2 & x_3 & x_4 & x_5 \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ x_4 & x_5 & x_6 & x_7 & x_8 & x_9 \\ x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} \end{matrix}$$

Windowing



Original data
600
400
200
Trend
400
300
200
Seasonality
1.4
1.2
1.0
0.8
Residuals
1.1
1.0
0.9
1950
1952
1954
1956
1958
1960

Windowing



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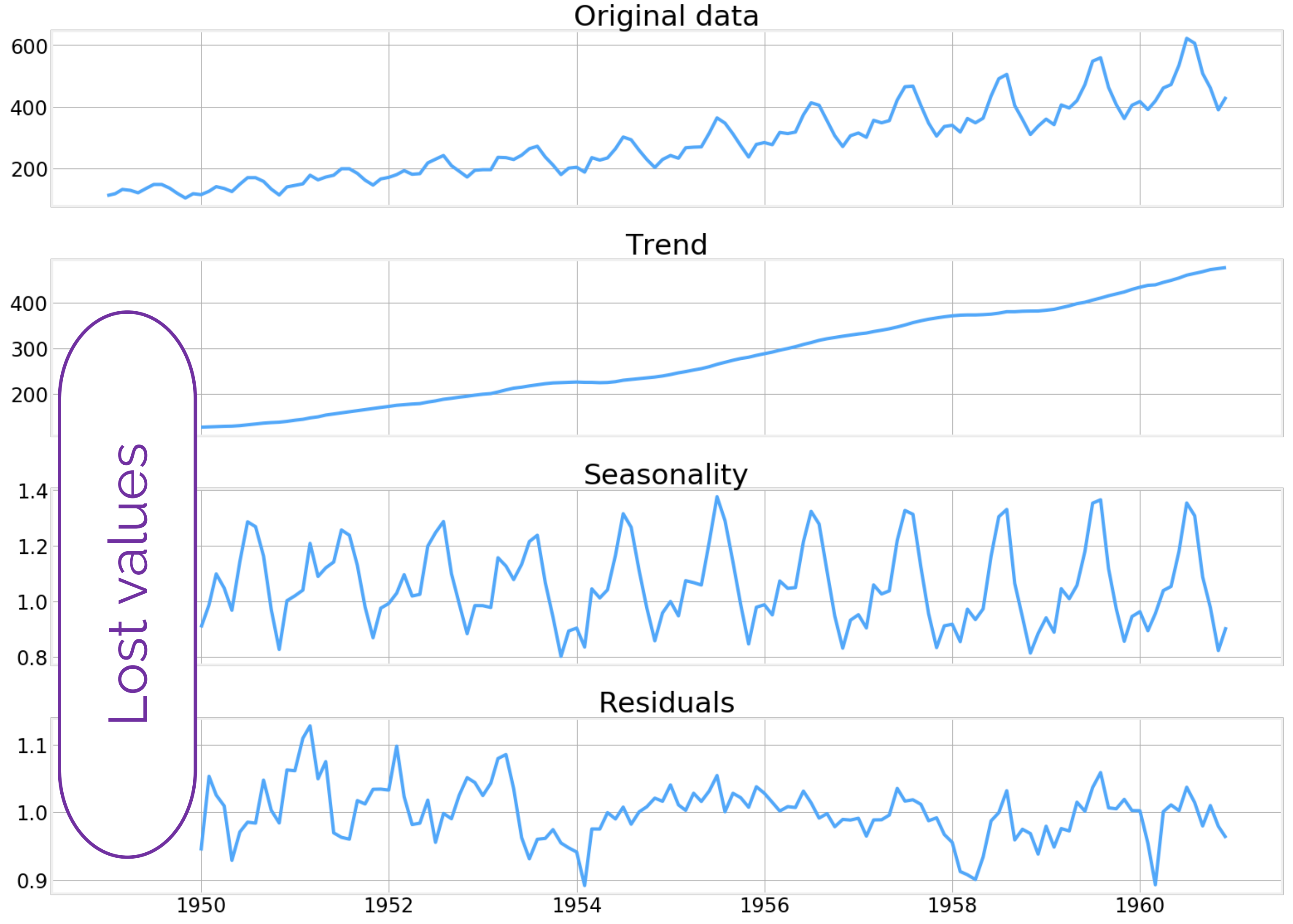
1950
1952
1954
1956
1958
1960
Lost values

Windowing

One common approach is to place all "lost values" at the beginning as it avoids "future leaking" when splitting the dataset



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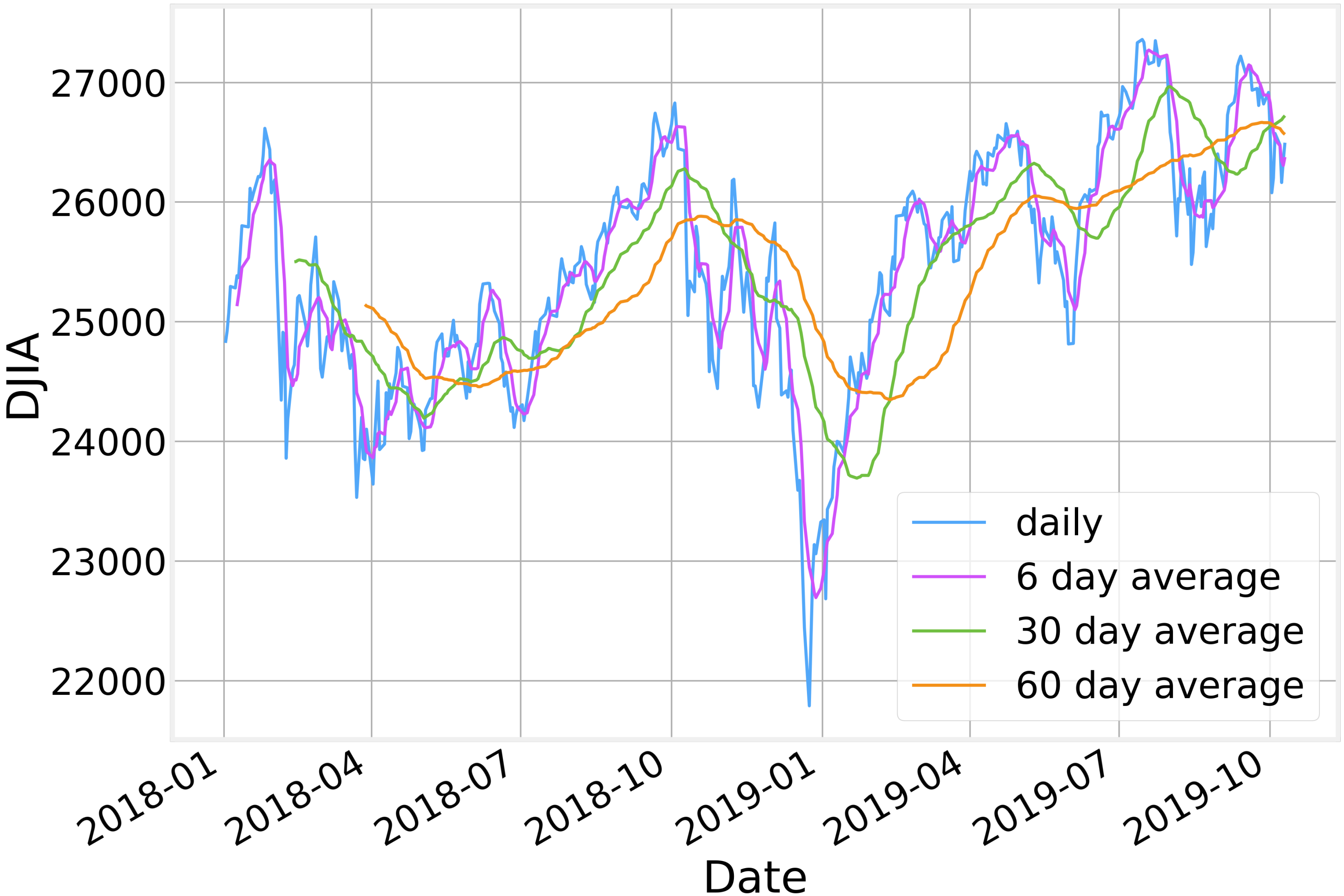
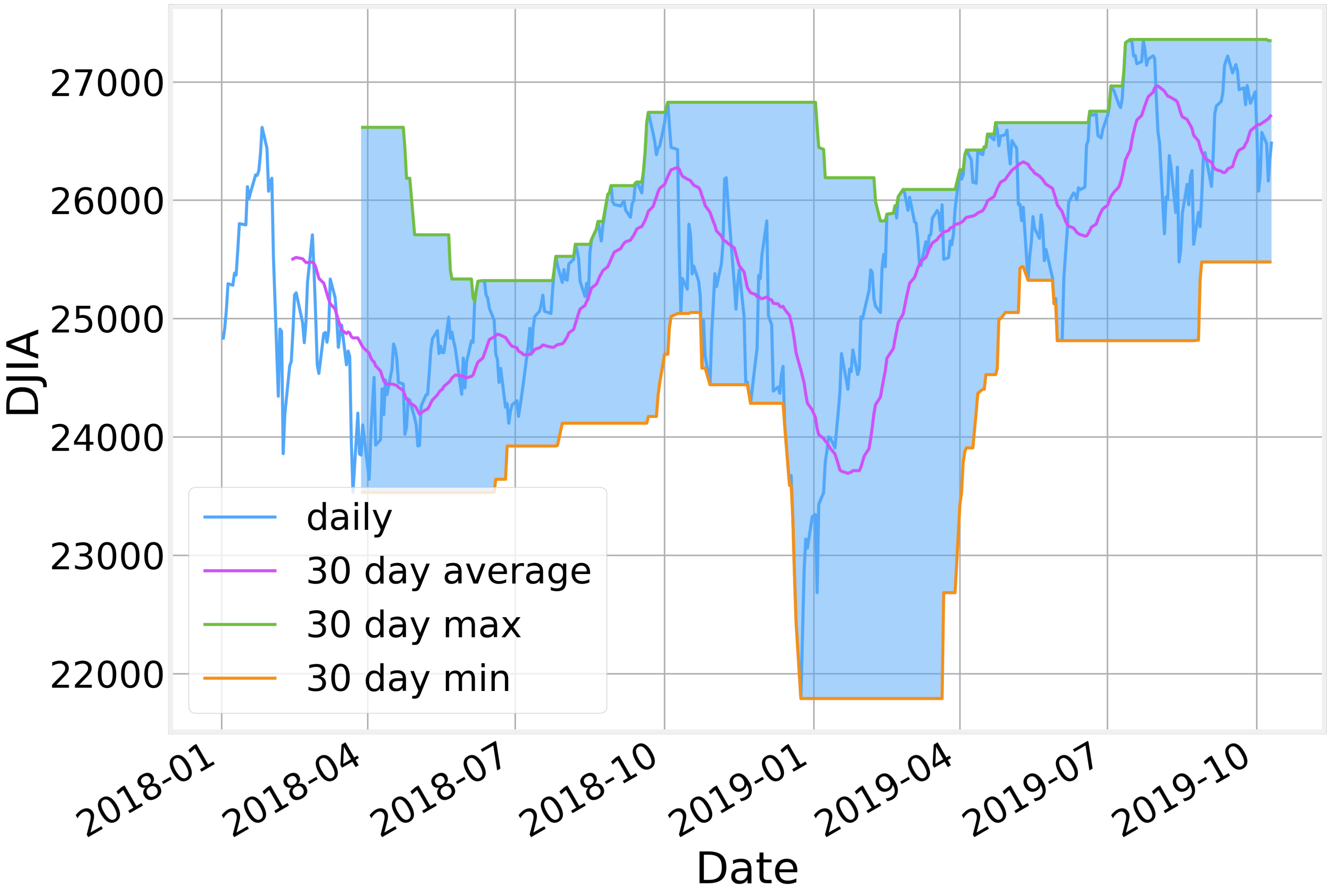
- Depending on the application we can choose to place the missing values in either or (or even both) extremes of the time interval

Running Values



Envelopes

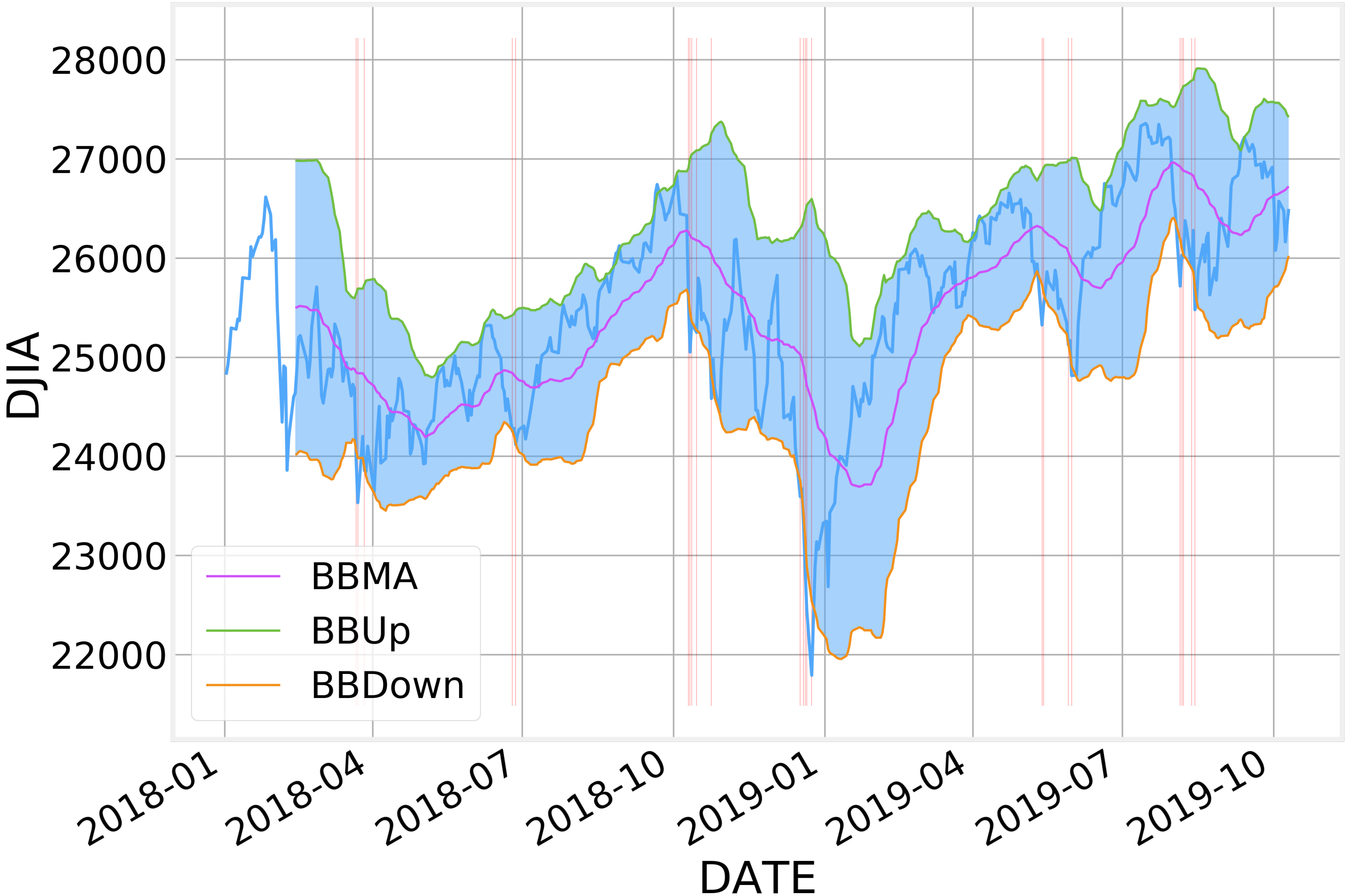


27000
26000
25000
24000
23000
22000
DJIA
daily
30 day average
30 day max
30 day min
2018-01
2018-04
2018-07
2018-10
2019-01
2019-04
2019-07
2019-10
Date

Bollinger Bands

- A common use for application for running values is the calculation of Bollinger Bands.
- Introduced by John Bollinger in the 1980s as a complement to more traditional time series technical analysis techniques.
- Bollinger Bands are defined by two components:
  - A $\boldsymbol{N}$ period moving average, μ_N
  - The area $\boldsymbol{K}$ standard deviations above and below the moving average $\mu_N \pm K\sigma_N$
- Both μ_N and σ_N are computed on a running window of size $\boldsymbol{N}$
- The values $\boldsymbol{N}$ and $\boldsymbol{K}$ are application specific. For stock trading, $\boldsymbol{N = 20}$ and $\boldsymbol{K = 2}$
- Whenever the time series steps out of the Bollinger Band that's a clear indication of a change in the temporal behavior.

Bollinger Bands



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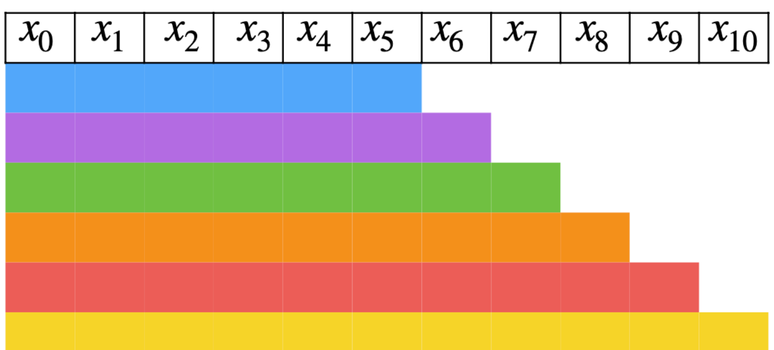

$$\begin{pmatrix} \alpha & & & & \\ \alpha(1-\alpha)^1 & \alpha & & & \\ \alpha(1-\alpha)^2 & \alpha(1-\alpha)^1 & \alpha & & \\ \vdots & \vdots & \vdots & \ddots & \\ \alpha(1-\alpha)^{n-1} & \alpha(1-\alpha)^{n-2} & \alpha(1-\alpha)^{n-3} & & \alpha \end{pmatrix}$$

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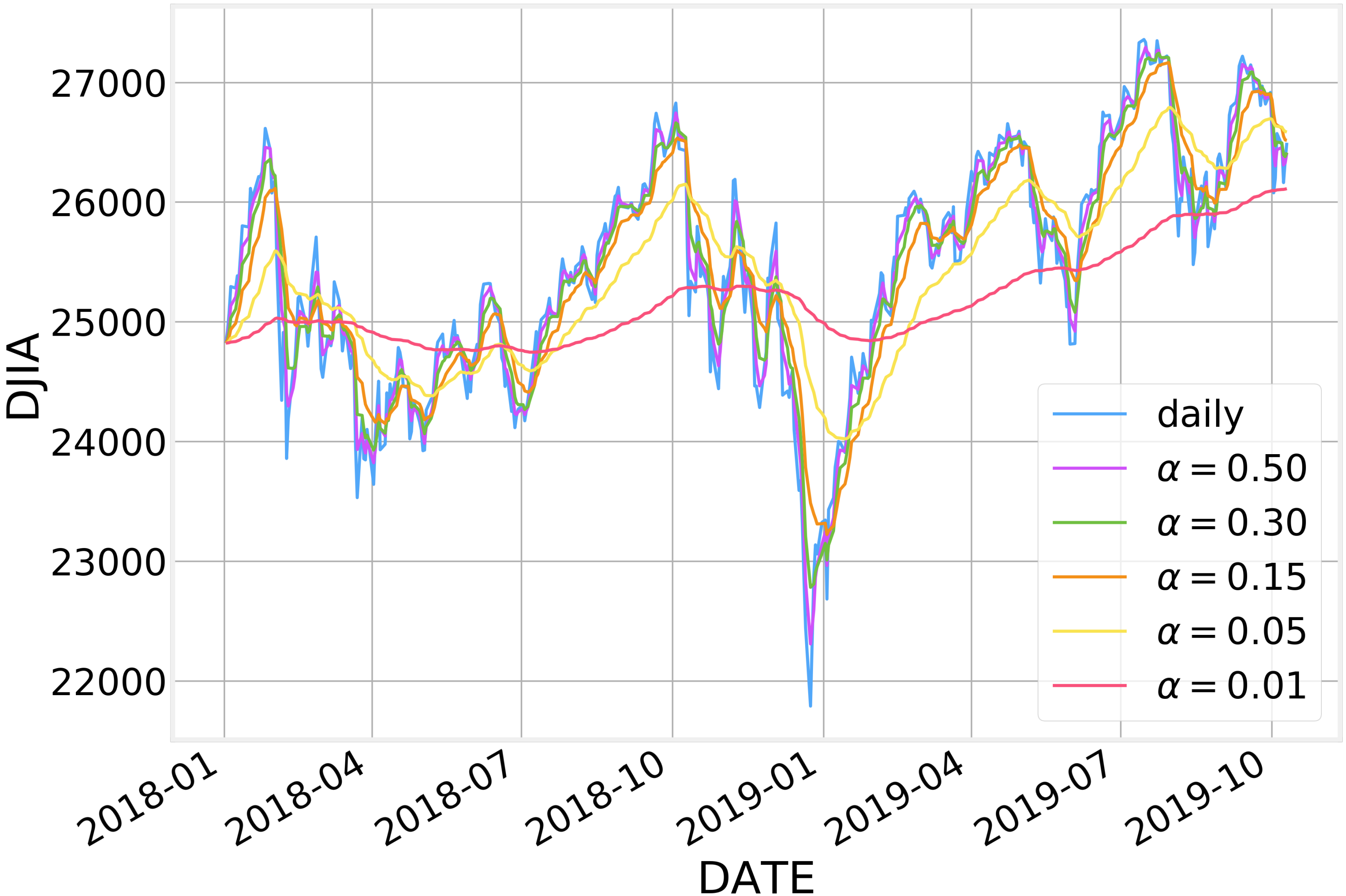

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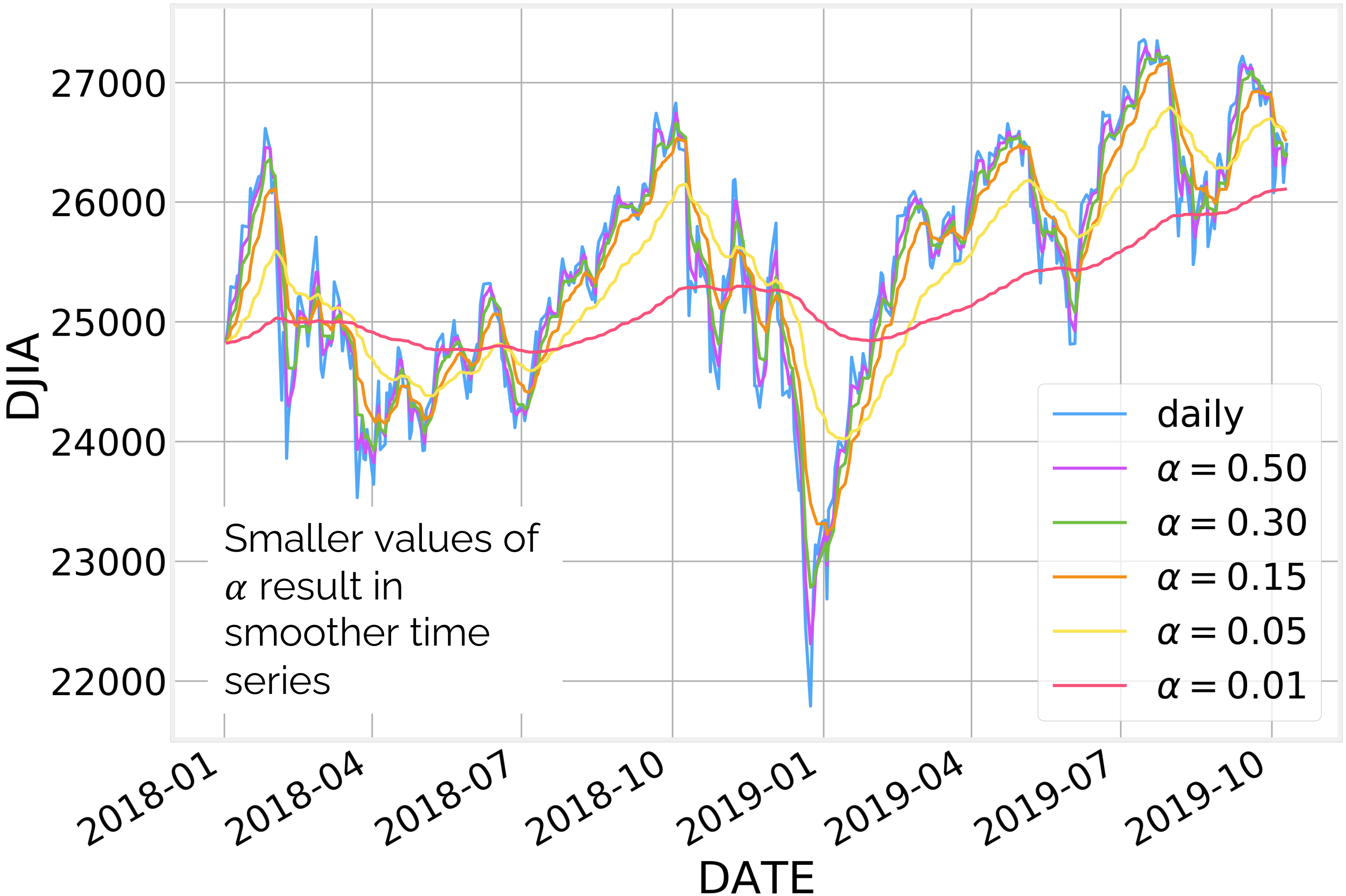

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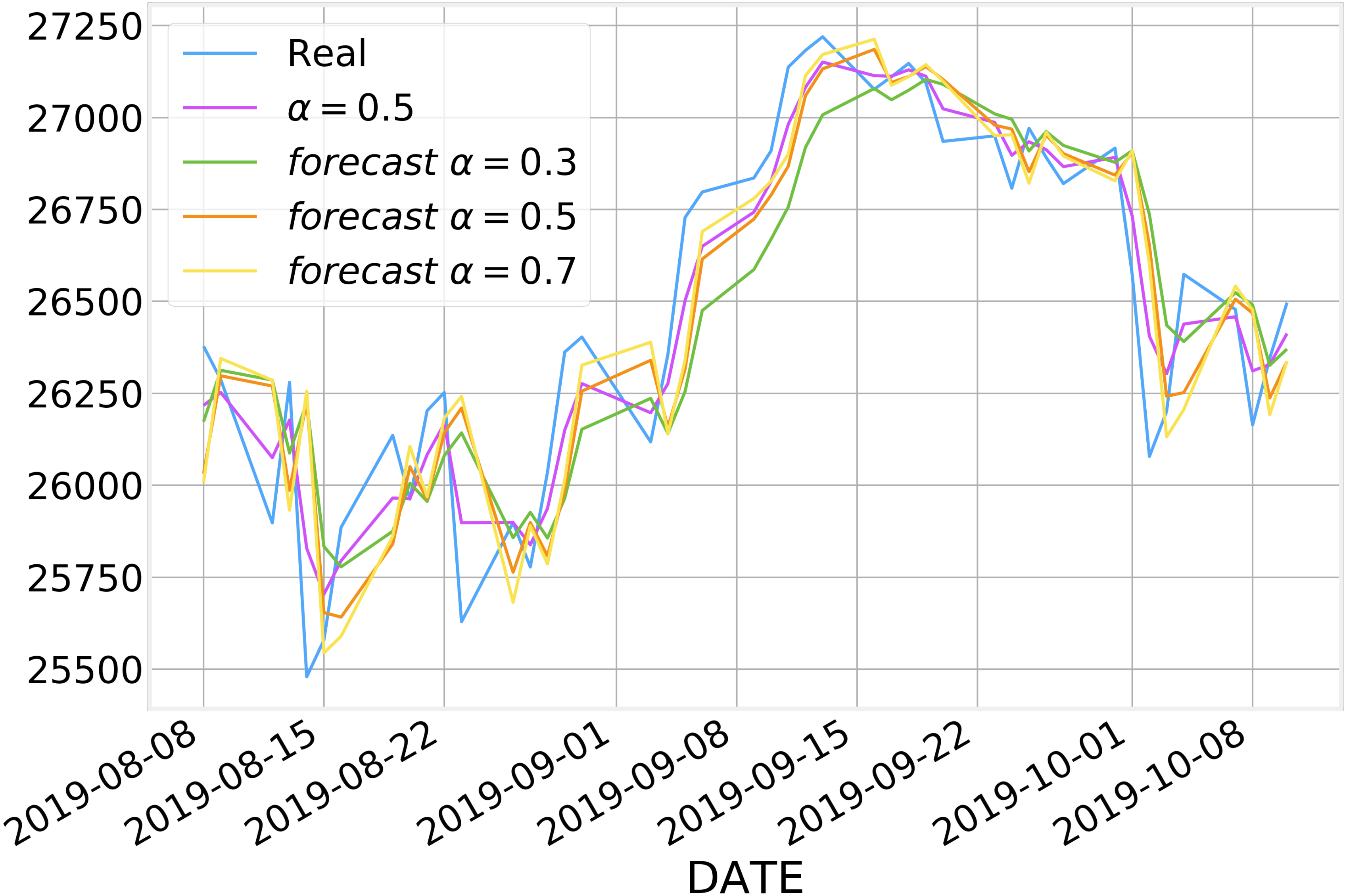
- Which we can consider to be a prediction on the value of x_{t+1}, based on the current value of z_t and some factor of out current error value $x_t - z_t$:

$$z_{t+1} = z_t + \alpha(x_t - z_t)$$

Forecasting



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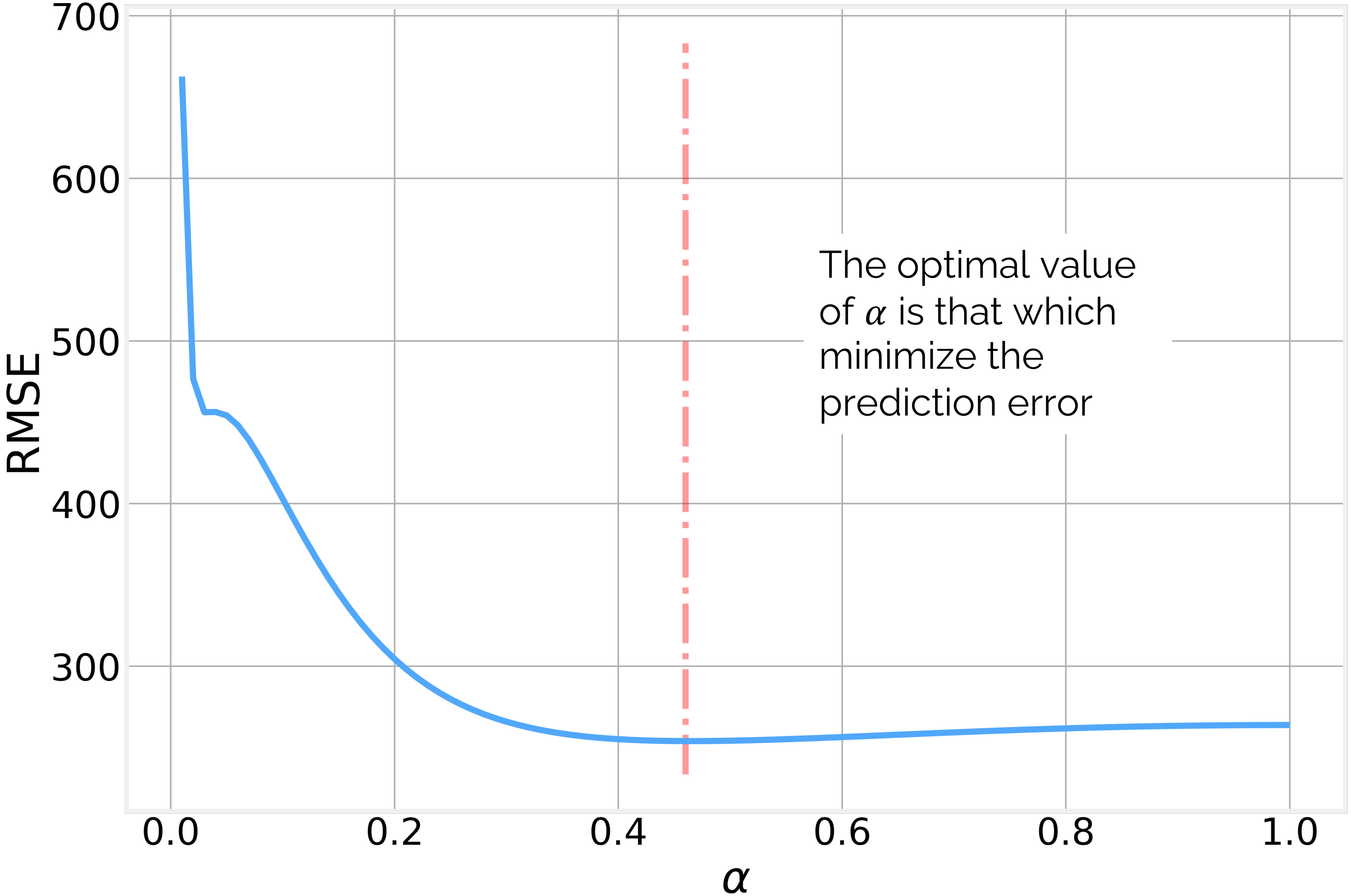

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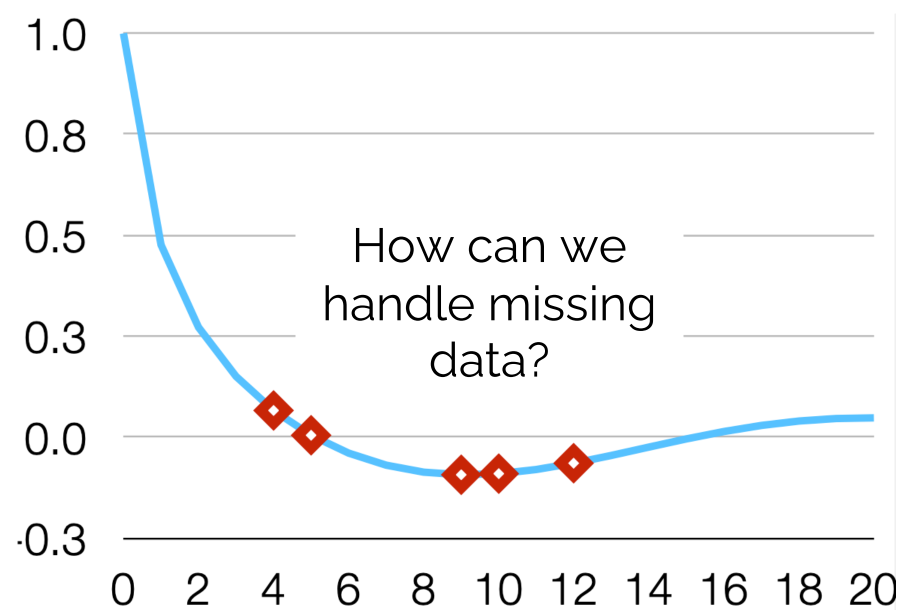

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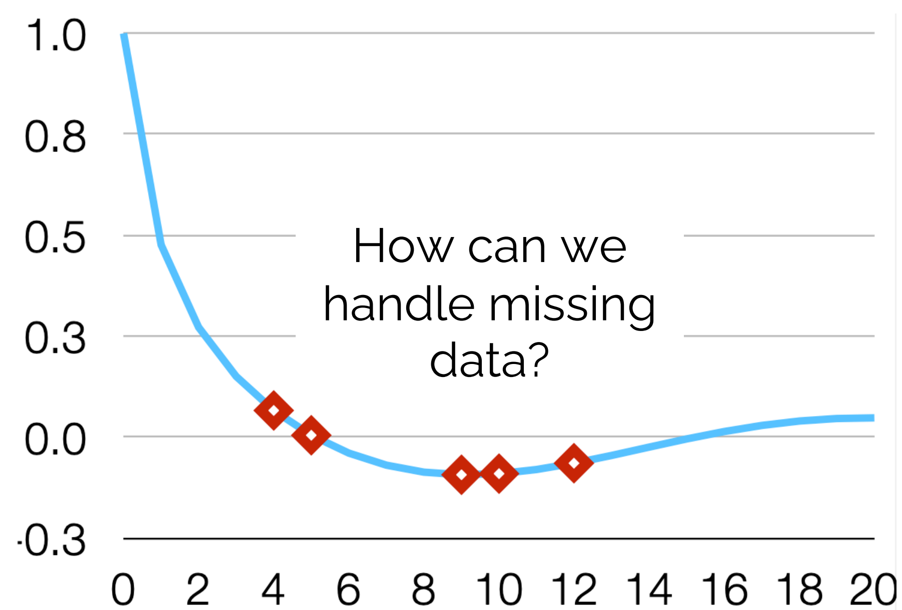

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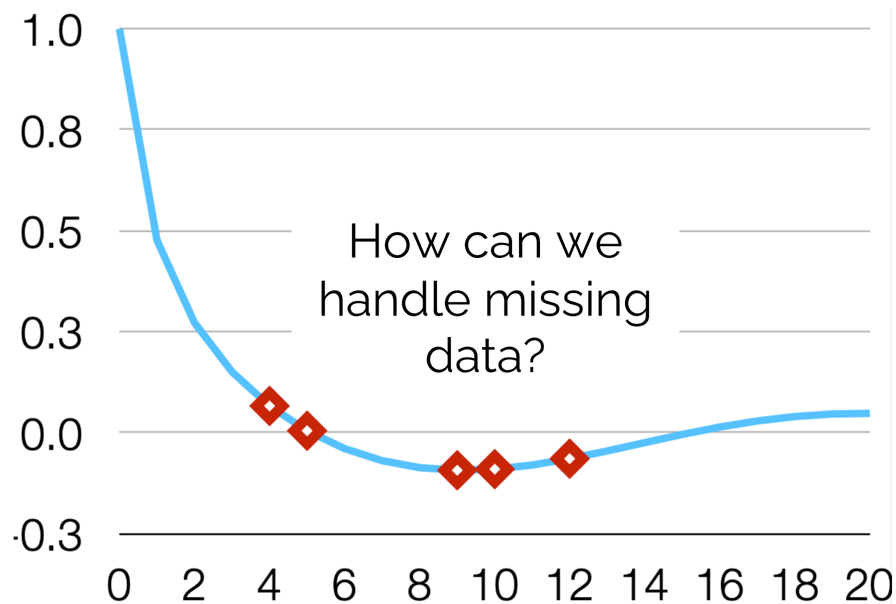

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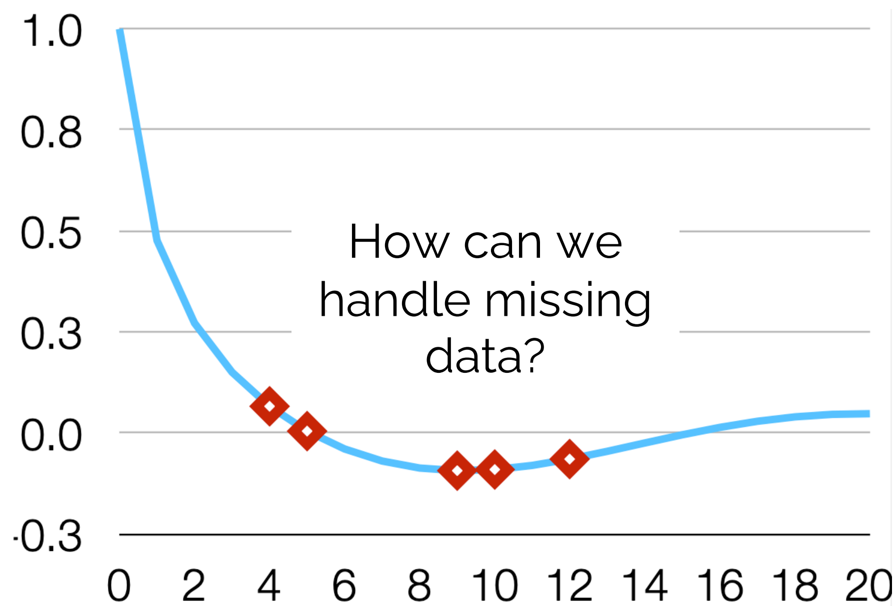

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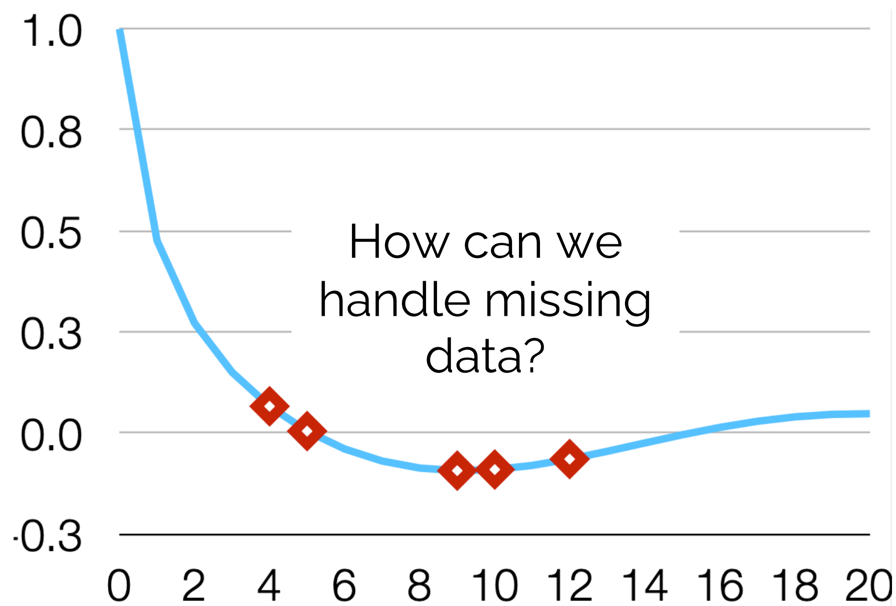

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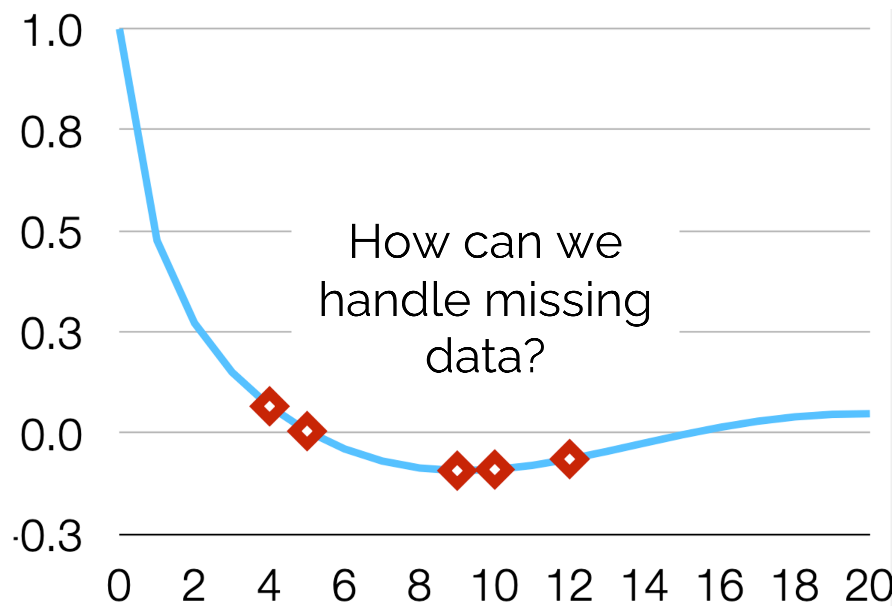

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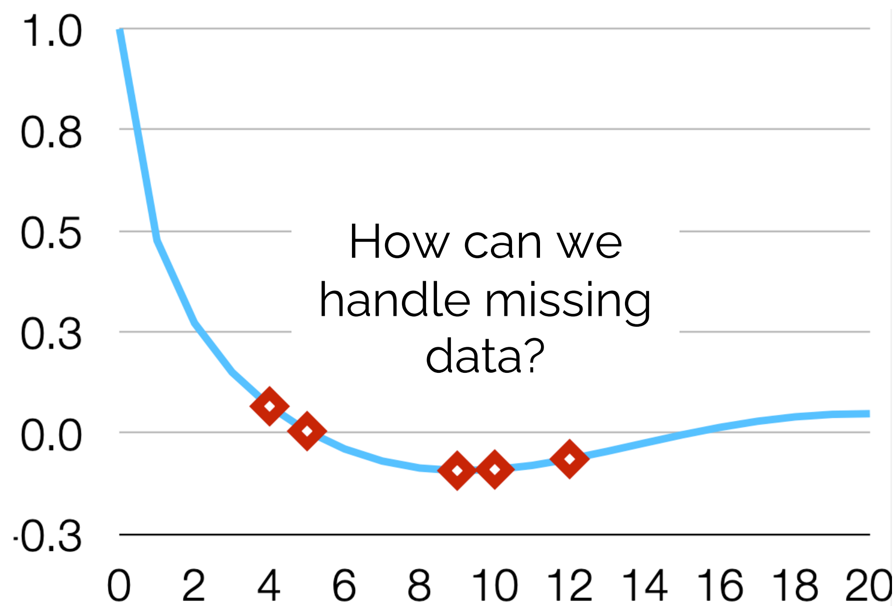

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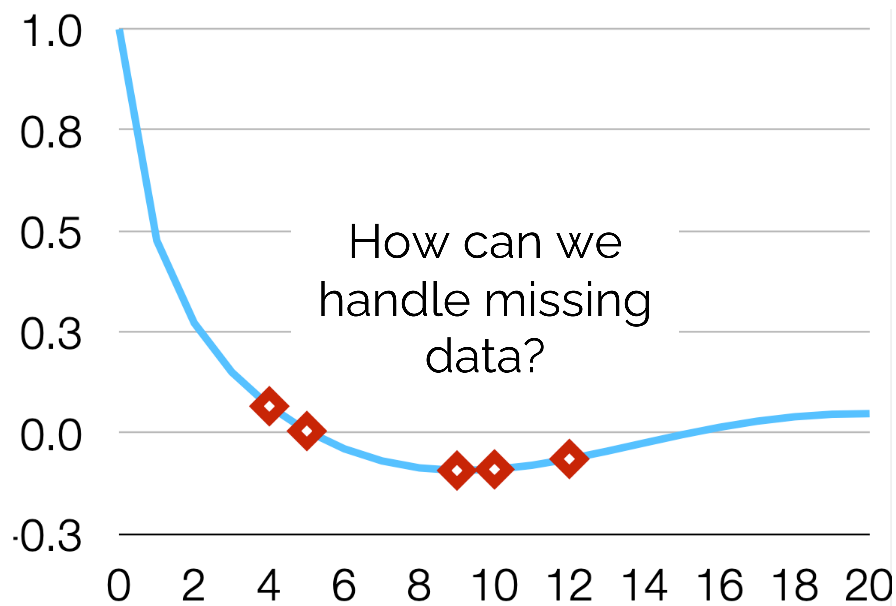

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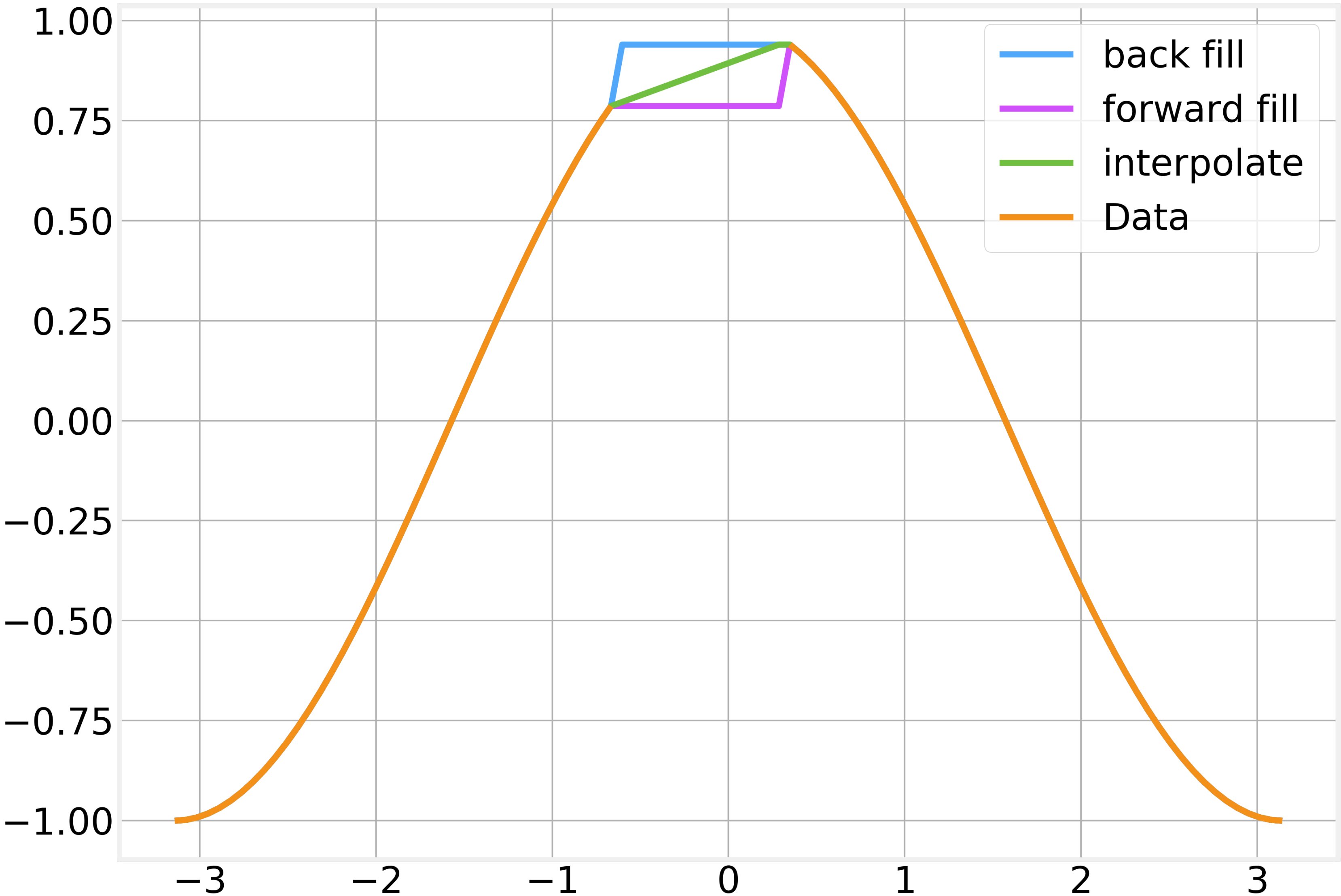

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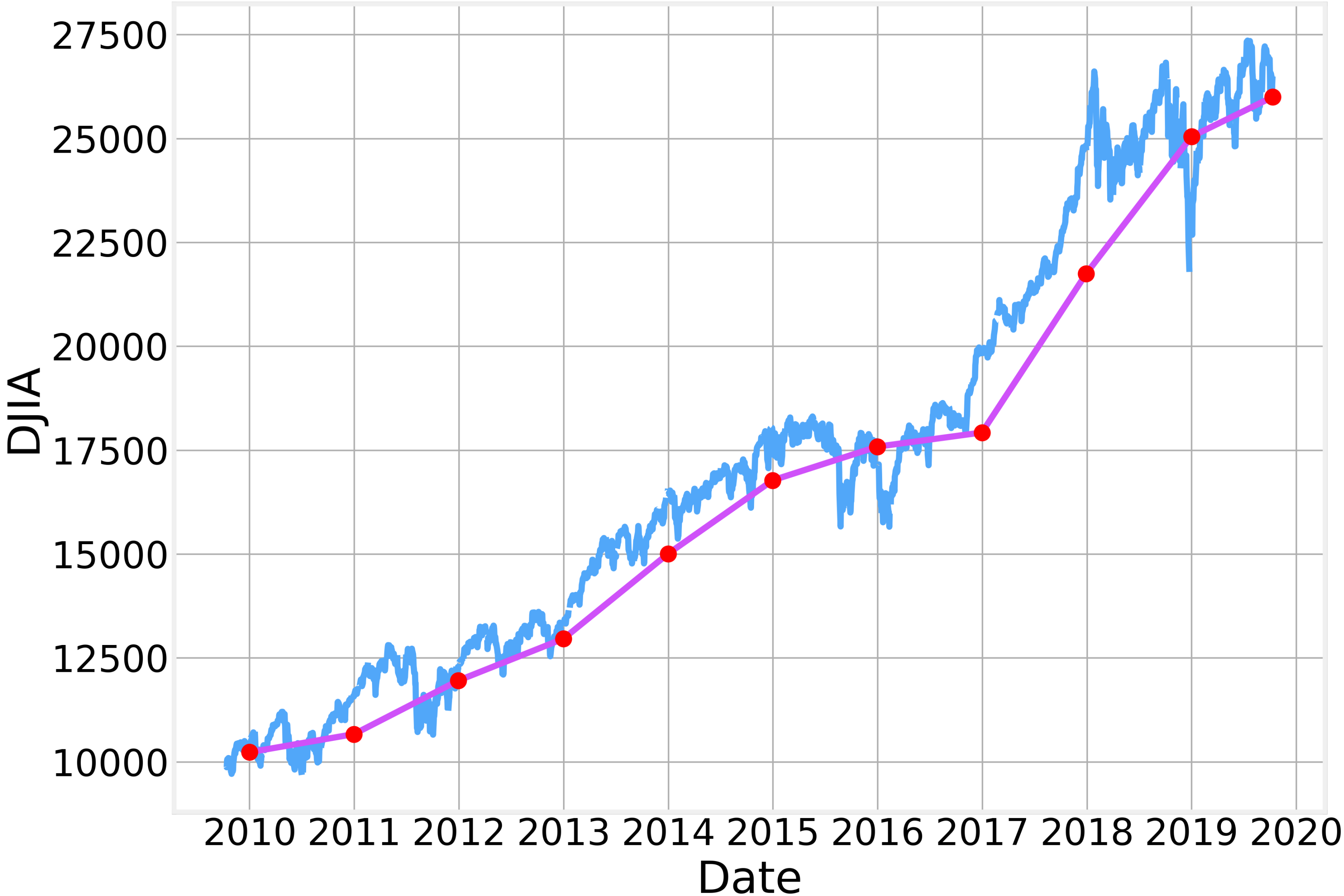

27500
25000
22500
20000
17500
15000
12500
10000
DJIA
2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020
Date

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