Time Series Analysis

INFO 523 - Lecture 13

Dr. Greg Chism

Lesson 3:
Correlations

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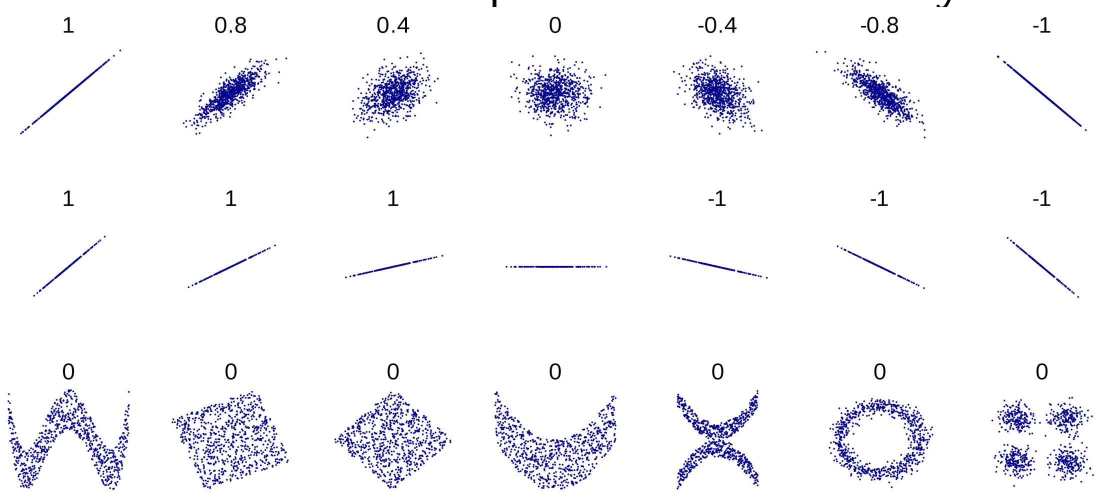

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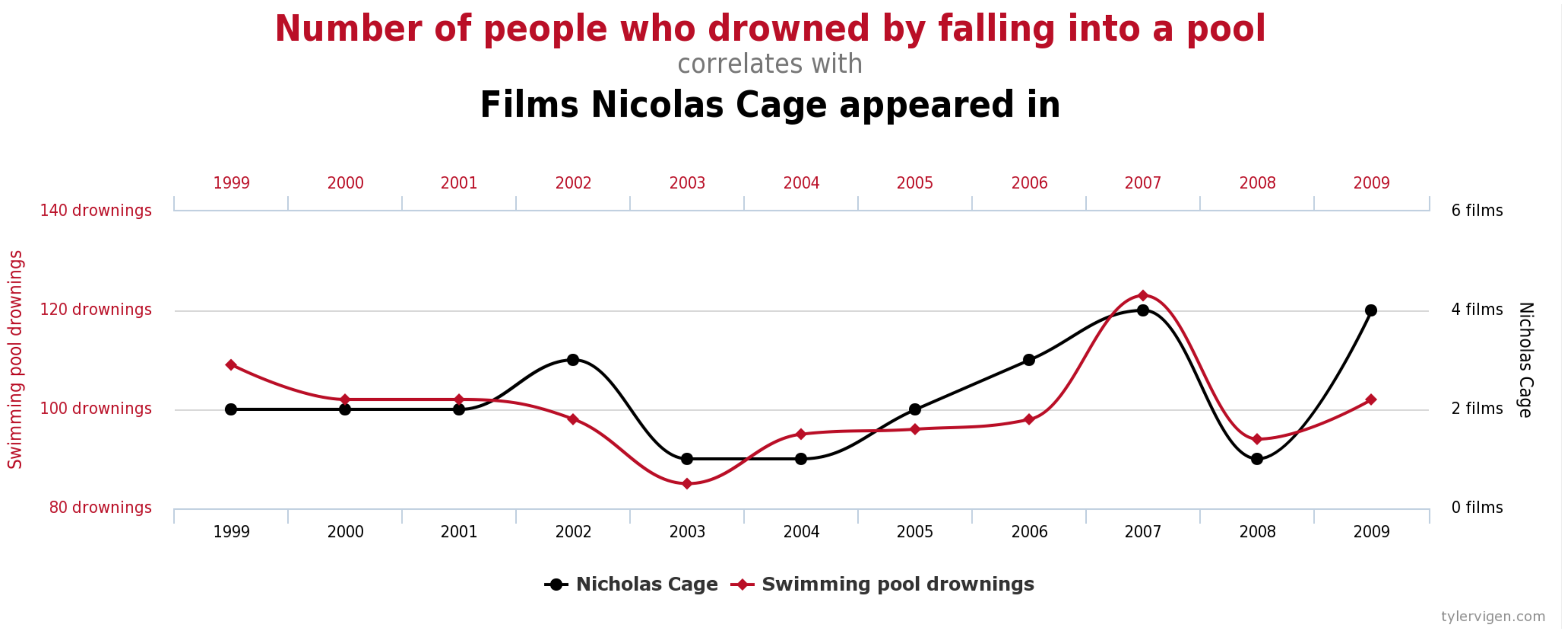

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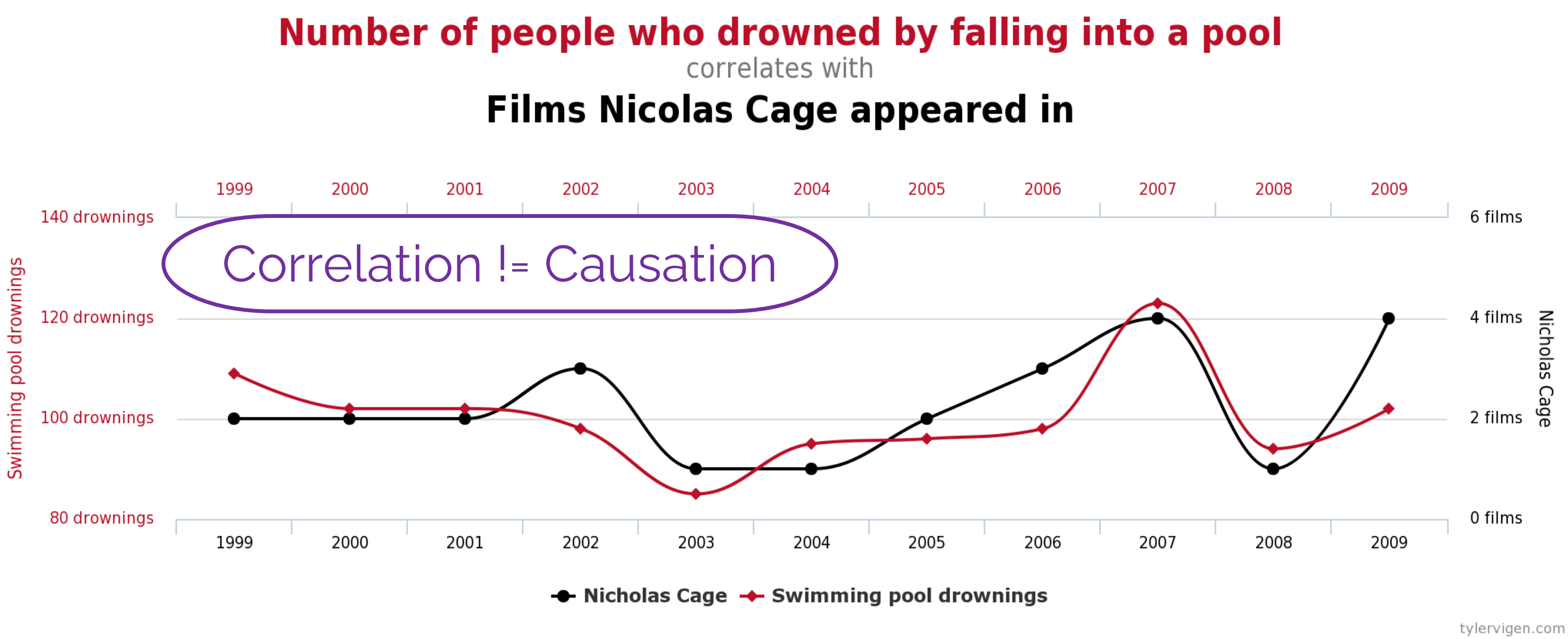

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- Correlation != Causation!
- Adding a trend to both series we immediately observe a significant correlation

The Pearson correlation of two trending series is overwhelmed by the trend

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https://en.wikipedia.org/wiki/Correlogram

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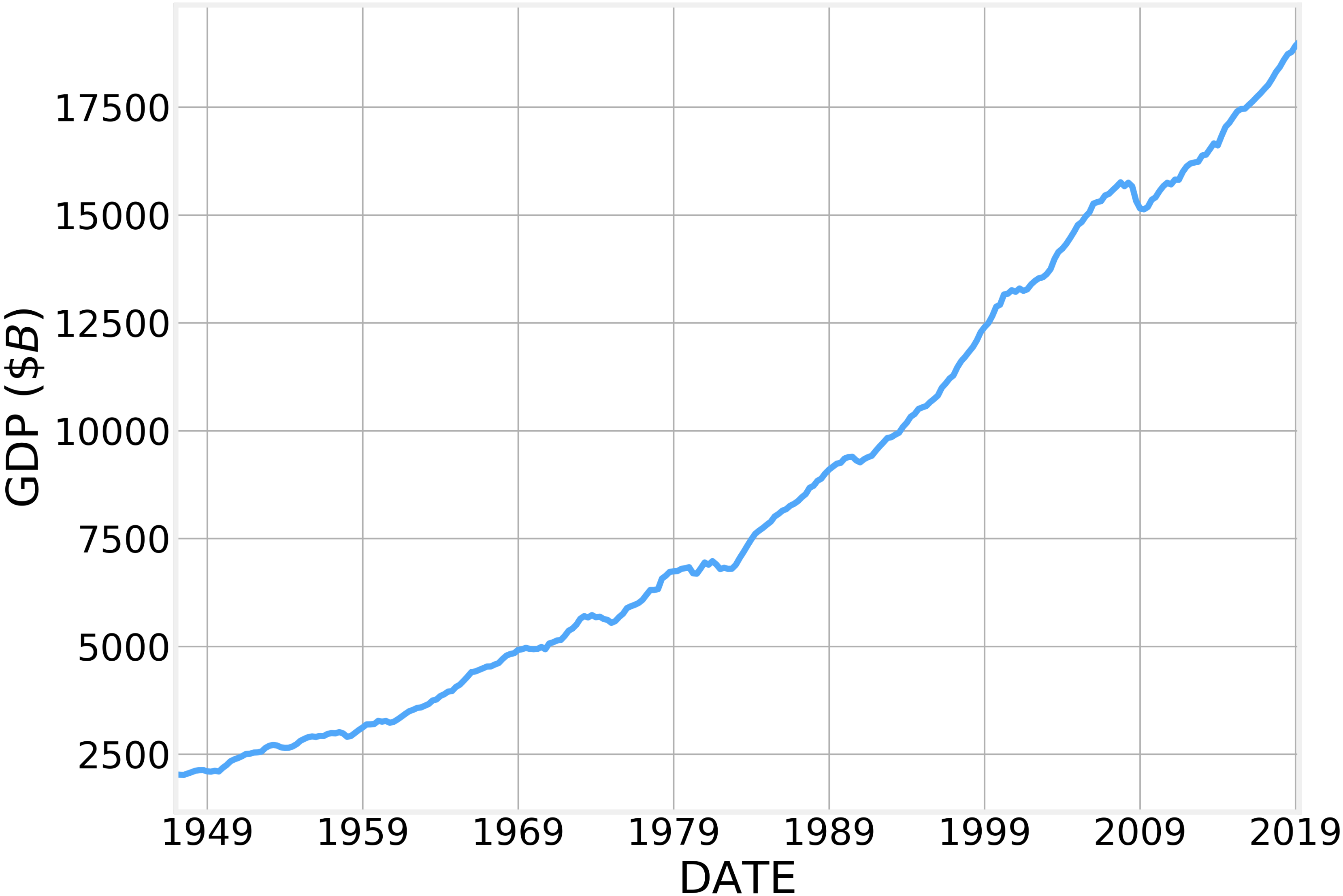

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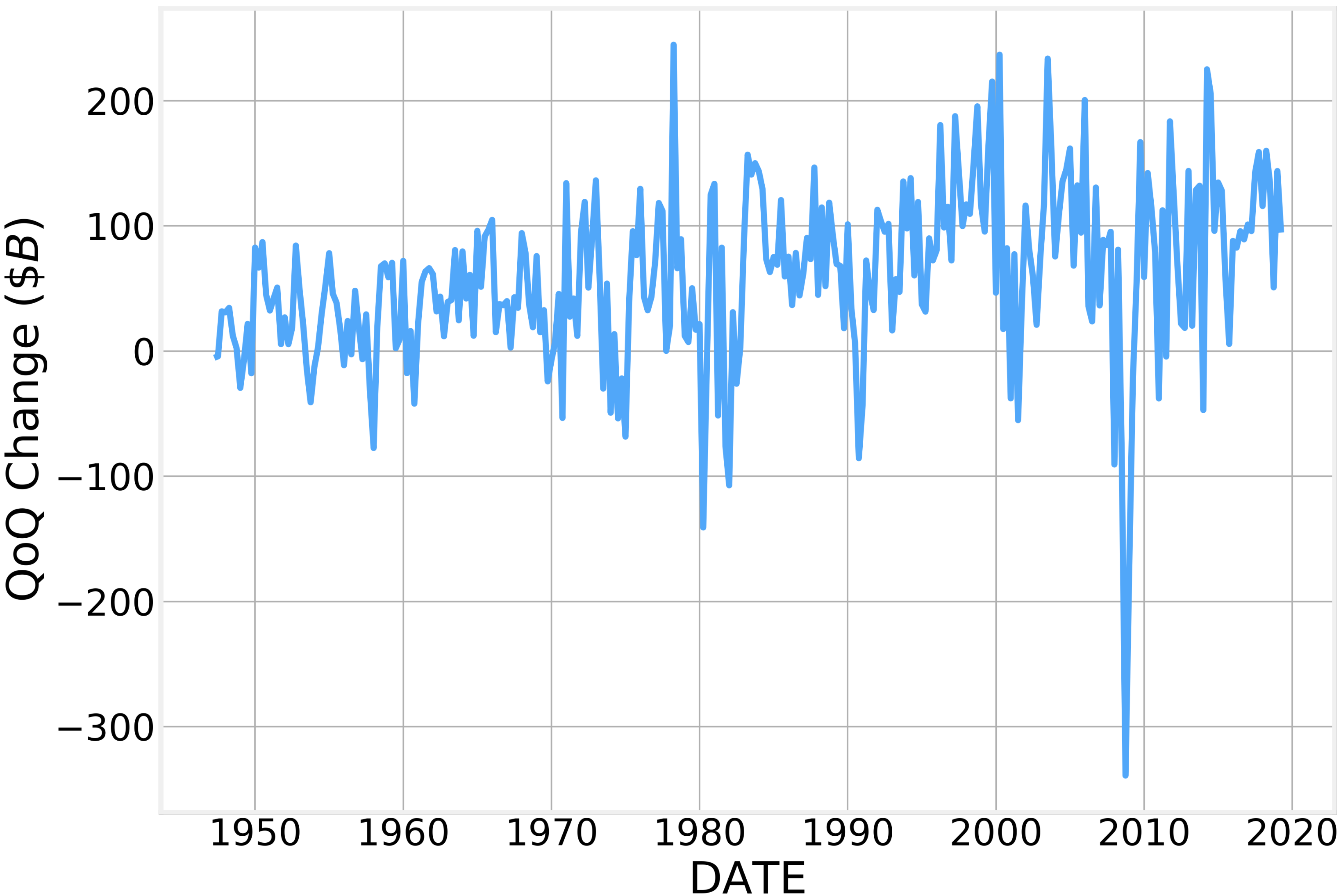

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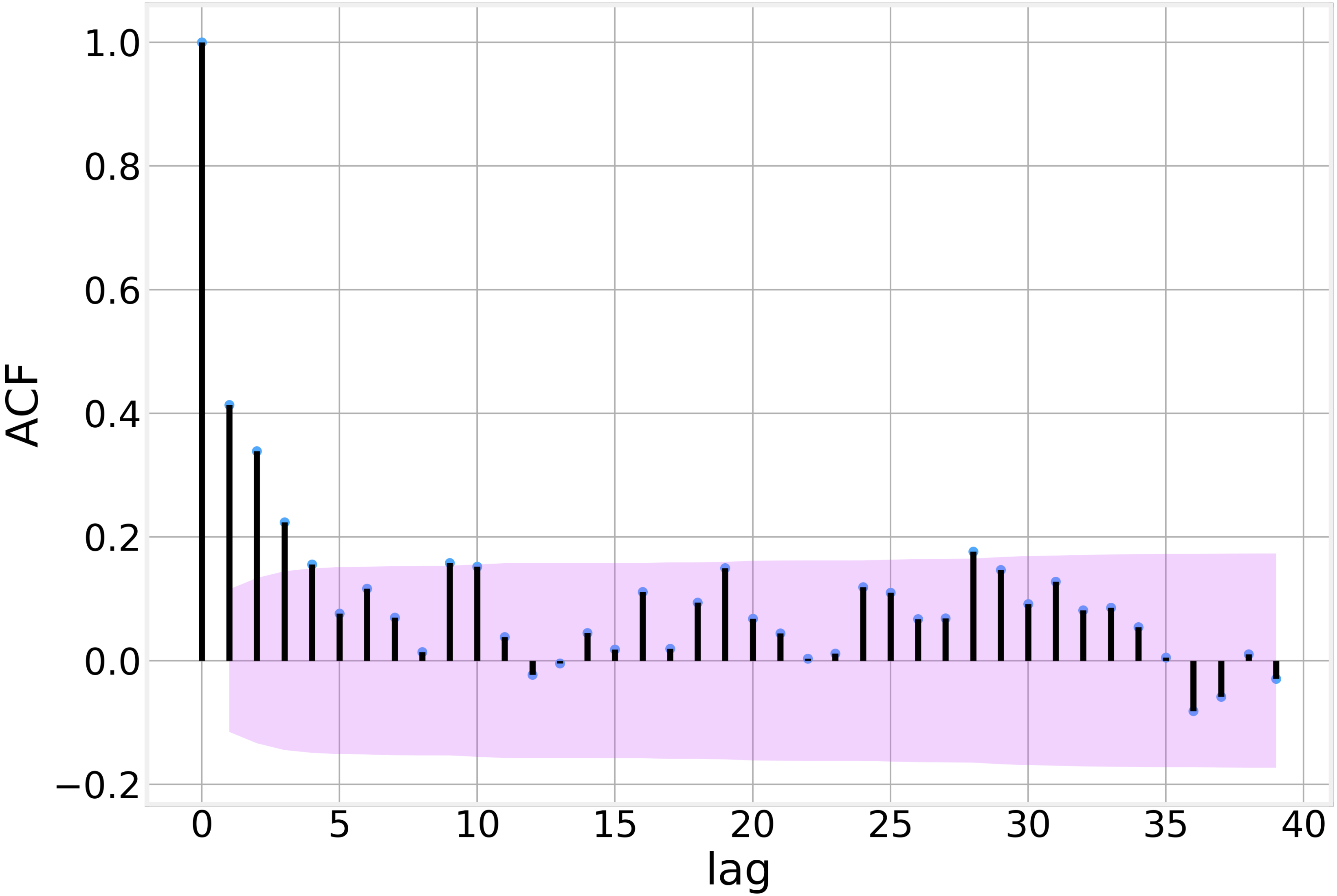

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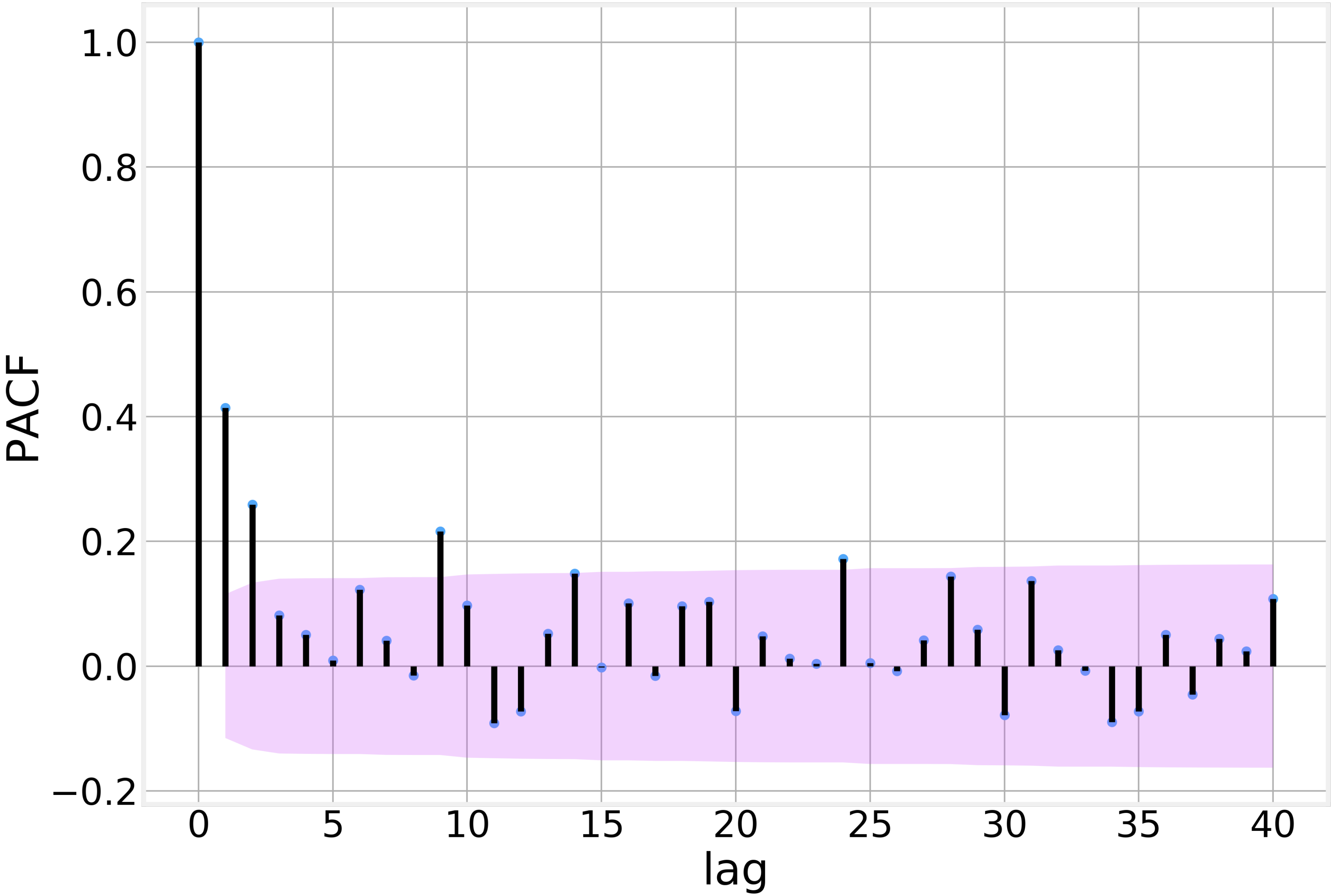

Code – Correlations